

# Composition of Functions

A **composite function** is when one function is plugged in to another function.

Composite Function Notation	In Words	Meaning
$(f \circ g)(x)$ Also written as $f(g(x))$	$f$ of $g(x)$	Plug the function $g(x)$ into the function $f(x)$
$(g \circ f)(x)$ Also written as $g(f(x))$	$g$ of $f(x)$	Plug the function $f(x)$ into the function $g(x)$

For domain of  $(f \circ g)(x)$ , find domain of  $(f \circ g)(x)$  and  $g(x)$

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**Example 1:** Given functions  $f(x) = \frac{4}{1-5x}$  and  $g(x) = \frac{1}{x}$

Find  $(f \circ g)(x)$

Step 1: Plug  $g(x)$  into  $f(x)$

$$f(x) = \frac{4}{1-5x} \quad g(x) = \frac{1}{x}$$

$$(f \circ g)(x) = \frac{4}{1-5\left(\frac{1}{x}\right)} = \frac{4}{1-\frac{5}{x}} = \left(\frac{4}{1-\frac{5}{x}}\right) \cdot \frac{x}{x} = \frac{4x}{x-\frac{5x}{x}} = \frac{4x}{x-5}$$

$$(f \circ g)(x) = \frac{4x}{x-5}$$

(multiply top and bottom by lowest common denominator to simplify complex fraction)

Step 2: Find domain

For  $(f \circ g)(x) = \frac{4x}{x-5}$ , denominator cannot be zero, so  $x \neq 5$ . In interval notation,  $(-\infty, 5) \cup (5, \infty)$

For  $g(x) = \frac{1}{x}$ , denominator cannot be zero, so  $x \neq 0$ . In interval notation,  $(-\infty, 0) \cup (0, \infty)$

So combining the domains, the domain is:  $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$

# Composition of Functions

**Example 2:** Given the functions  $f(x) = x + 2$  and  $g(x) = 4 - x^2$

Find  $(f \circ g)(4)$

**Method 1:** Plug in right to left  $(f \circ g)(4)$

This can also be seen as  $f(g(4))$

Step 1: Plug 4 into  $g(x)$

$$g(4) = 4 - (4)^2 = 4 - 16 = -12$$

$$g(4) = -12$$

Step 2: Plug  $g(4)$  into  $f(x)$

$$f(g(4)) = f(-12) = (-12) + 2 = -10$$

Therefore,  $(f \circ g)(4) = -10$

**Method 2:** Find  $(f \circ g)(x)$  first, then plug 4 into the result.

Step 1: Plug  $g(x)$  into  $f(x)$

$$f(g(x)) = (4 - x^2) + 2 = 4 - x^2 + 2 = 6 - x^2$$

$$(f \circ g)(x) = 6 - x^2$$

Step 2: Plug 4 into  $(f \circ g)(x)$

$$(f \circ g)(4) = 6 - (4)^2 = 6 - 16 = -10$$

Therefore,  $(f \circ g)(4) = -10$

We get the same answer as method 1.

# Decomposition of Functions

To **decompose** a function is to split a function into an “inner” function and an “outer function”

Note: There can be several ways to decompose the same function.

**Example:** Find two functions  $f$  and  $g$  such that  $(f \circ g)(x) = h(x)$

Given the function:  $h(x) = \sqrt{x^3 + 1}$

**One option is:**  $f(x) = \sqrt{x}$  and  $g(x) = x^3 + 1$

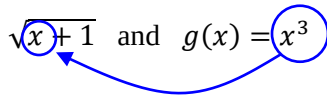
Since plugging in we get:

$$f(x) = \sqrt{x} \quad \text{and} \quad g(x) = x^3 + 1$$


$$(f \circ g)(x) = \sqrt{x^3 + 1} = h(x)$$

**Another option is:**  $f(x) = \sqrt{x + 1}$  and  $g(x) = x^3$

Since plugging in we get:

$$f(x) = \sqrt{x + 1} \quad \text{and} \quad g(x) = x^3$$


$$(f \circ g)(x) = \sqrt{x^3 + 1} = h(x)$$