Composition of Functions

A composite function is when one function is plugged in to another function.

Composite Function Notation	In Words	Meaning
$(f \circ g)(x)$ Also written as $f(g(x))$	f of $g(x)$	Plug the function $g(x)$ into the function $f(x)$
$(g \circ f)(x)$ Also written as $g(f(x))$	g of $f(x)$	Plug the function $f(x)$ into the function $g(x)$

For domain of $(f \circ g)(x)$, find domain of $(f \circ g)(x)$ and $g(x)$

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Example 1: Given functions $f(x) = \dfrac{4}{1-5x}$ and $g(x) = \dfrac{1}{x}$

Find $(f \circ g)(x)$

Step 1: Plug $g(x)$ into $f(x)$ $\quad f(x) = \dfrac{4}{1-5x} \quad\quad g(x) = \dfrac{1}{x}$

$$(f \circ g)(x) = \frac{4}{1-5\left(\frac{1}{x}\right)} = \frac{4}{1-\frac{5}{x}} = \left(\frac{4}{1-\frac{5}{x}}\right) \cdot \frac{x}{x} = \frac{4x}{x-\frac{5x}{x}} = \frac{4x}{x-5}$$

(multiply top and bottom by lowest common denominator to simplify complex fraction)

$$(f \circ g)(x) = \frac{4x}{x-5}$$

Step 2: Find domain

For $(f \circ g)(x) = \dfrac{4x}{x-5}$, denominator cannot be zero, so $x \neq 5$. In interval notation, $(-\infty, 5) \cup (5, \infty)$

For $g(x) = \dfrac{1}{x}$, denominator cannot be zero, so $x \neq 0$. In interval notation, $(-\infty, 0) \cup (0, \infty)$

So combining the domains, the domain is: $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$

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Example 2: Given the functions $f(x) = x + 2$ and $g(x) = 4 - x^2$

Find $(f \circ g)(4)$

Method 1: Plug in right to left $(f \circ g)(4)$

This can also be seen as $f\big(g(4)\big)$

Step 1: Plug 4 into $g(x)$

$$g(4) = 4 - (4)^2 = 4 - 16 = -12$$

$$g(4) = -12$$

Step 2: Plug $g(4)$ into $f(x)$

$$f\big(g(4)\big) = f(-12) = (-12) + 2 = -10$$

Therefore, $(f \circ g)(4) = -10$

Method 2: Find $(f \circ g)(x)$ first, then plug 4 into the result.

Step 1: Plug $g(x)$ into $f(x)$

$$f\big(g(x)\big) = (4 - x^2) + 2 = 4 - x^2 + 2 = 6 - x^2$$

$$(f \circ g)(x) = 6 - x^2$$

Step 2: Plug 4 into $(f \circ g)(x)$

$$(f \circ g)(4) = 6 - (4)^2 = 6 - 16 = -10$$

Therefore, $(f \circ g)(4) = -10$

We get the same answer as method 1.

Decomposition of Functions

To decompose a function is to split a function into an "inner" function and an "outer function"
Note: There can be several ways to decompose the same function.

Example: Find two functions f and g such that $(f \circ g)(x) = h(x)$

Given the function: $h(x) = \sqrt{x^3 + 1}$

One option is: $f(x) = \sqrt{x}$ and $g(x) = x^3 + 1$

Since plugging in we get:

$f(x) = \sqrt{x}$ and $g(x) = x^3 + 1$

$$(f \circ g)(x) = \sqrt{x^3 + 1} = h(x)$$

Another option is: $f(x) = \sqrt{x + 1}$ and $g(x) = x^3$

Since plugging in we get:

$f(x) = \sqrt{x + 1}$ and $g(x) = x^3$

$$(f \circ g)(x) = \sqrt{x^3 + 1} = h(x)$$