Exponential Functions

Exponential functions have the form $\boldsymbol{f(x) = b^x}$ (note that x is in the exponent)
where b is a positive constant number called the base or growth factor
and x is a variable.

The general form of an exponential function can also be written as: $\boldsymbol{f(x) = a(b)^x}$
where the a represents the y-intercept, which is often called the starting or initial value.

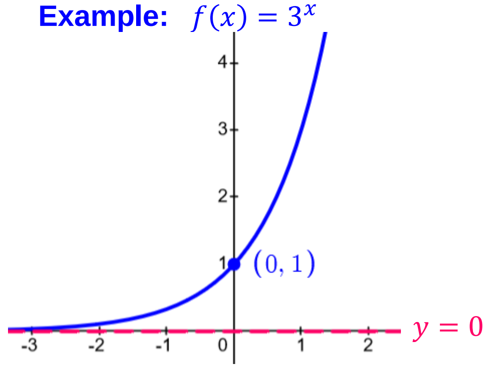
Examples of exponential functions:

$f(x) = 2^x$ $\quad y = 5(1.26)^t$ $\quad g(x) = \left(\frac{1}{3}\right)^{x-1}$ $\quad A = e^{-x}$

Properties of Exponential Functions

For $f(x) = b^x$	
Domain:	$(-\infty, \infty)$
Range:	$(0, \infty)$
x-intercepts:	None
y-intercepts:	$(0, 1)$
Horizontal Asymptote:	$y = 0$

Example: $f(x) = 3^x$



Formulas:

Compound Interest Formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = Future Amount, P = Principle (starting amount)
r = rate, n = compounding periods
t= time in years

Compounding Period	n
Annually	1
Semiannually	2
Quarterly	4
Monthly	12
Daily	365

Continuous Compound Interest:

$$A = Pe^{rt}$$

Exponential Growth:

$$y = y_0 e^{kt}$$

Half Life:

$$h = \frac{\ln 2}{k}$$

Exponential Decay:

$$y = y_0 e^{-kt}$$

Log Rules and Properties

$\log x = \log_{10} x$

$\ln x = \ln_e x$

$\log_b 1 = 0$

$\ln 1 = 0$

$\log 10 = 1$

$\ln e = 1$

$\log_b b^x = x$

$b^{\log_b x} = x$

Change of base formula:	**Example:**
$\log_a x = \dfrac{\log x}{\log a}$	$\log_6 5 = \dfrac{\log 5}{\log 6}$

Examples:

$\log 10^2 = 2$

$\ln e^3 = 3$

$\log_3 1 = 0$

$\log_5 5^4 = 4$

$\log_5 \frac{1}{5} = \log_5 5^{-1} = -1$

$\log_4 16 = \log_4 4^2 = 2$

$7^{\log_7 9} = 9$

Log Rules for Expanding and Condensing Logarithms:

Product Rule:	$\log_a xy = \log_a x + \log_a y$	(multiplication changes to addition)
Quotient Rule:	$\log_a \frac{x}{y} = \log_a x - \log_a y$	(division changes to subtraction)
Power Rule:	$\log_a x^r = r \log_a x$	(exponent moves to the front)

Example 1: Expand the expression

$\log_2(x^4 y^3)$

$$\log_2(x^4 y^3) = \log_2 x^4 + \log_2 y^3$$

$$= 4\log_2 x + 3\log_2 y$$

Example 2: Condense the expression into a single logarithm

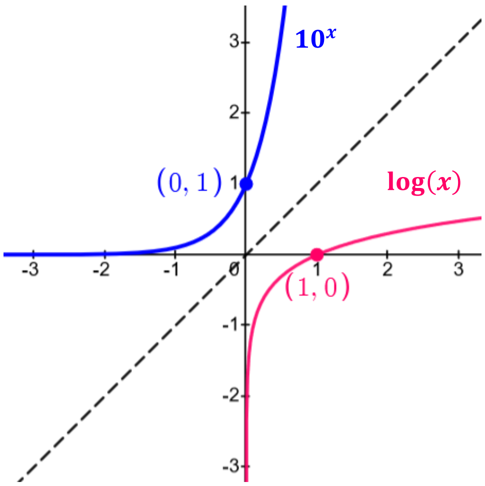
$5\log_3 x - \log_3 y$

$$5\log_3 x - \log_3 y = \log_3 x^5 - \log_3 y$$

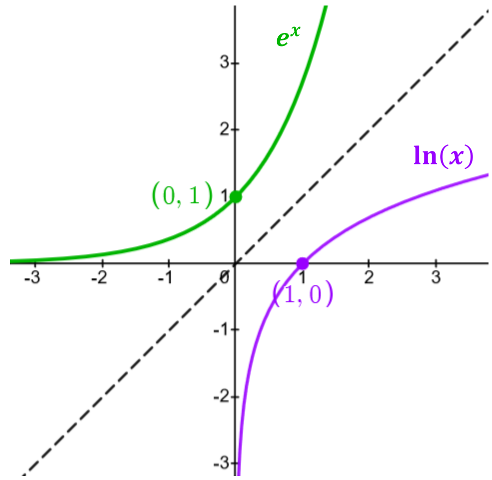
$$= \log_3 \frac{x^5}{y}$$

Logarithms

Exponential and Logarithmic functions are inverses of each other.



$\log(x)$ is base 10: $\log_{10}(x)$



$\ln(x)$ (natural log) is base e: $\ln_e(x)$

Converting between Logarithmic and Exponential Form:

$$b^x = a \leftrightarrow \log_b a = x$$

Example: $6^x = 28 \leftrightarrow \log_6 28 = x$

One way to convert between log and exponential is simply rearranging the parts like shown above.

The examples below show the steps to converting by using log properties.

Example 1: Convert to a logarithmic equation:

$$6^x = 28$$

Step 1: Take log base 6 of both sides

$$\log_6 6^x = \log_6 28$$

$$\cancel{\log_6} \cancel{6}^x = \log_6 28$$

log base 6 and 6 cancel each other out

$$x = \log_6 28$$

Example 2: Convert to a logarithmic equation:

$$e^x = 20$$

Step 1: Take natural log of both sides

$$\ln e^x = \ln 20$$

$$\cancel{\ln} \cancel{e}^x = \ln 20$$

ln and e cancel each other out

$$x = \ln 20$$

Example 3: Convert to an exponential equation:

$$y = \log_4 7$$

Step 1: Introduce 4 to both sides, raising everything to exponents

$$4^y = 4^{\log_4 7}$$

$$4^y = \cancel{4}^{\cancel{\log_4} 7}$$

4 and log base 4 cancel each other out

$$4^y = 7$$

Example 4: Convert to an exponential equation

$$\ln 12 = 2.4849$$

Step 1: Introduce e to both sides, raising everything to exponents

$$e^{\ln 12} = e^{2.4849}$$

$$\cancel{e}^{\cancel{\ln} 12} = e^{2.4849}$$

ln and e cancel each other out

$$12 = e^{2.4849}$$

Solve Logarithmic Equations

Example: Solve $\log_3(x+4) + \log_3(x-4) = 2$

Step 1: Write as a single logarithm

$$\log_3\big((x+4)(x-4)\big) = 2$$

$$\log_3(x^2 - 16) = 2$$

Step 2: Cancel the logarithm by converting to exponential form

$$3^{\log_3(x^2-16)} = 3^2$$ the 3 and log base 3 cancel each other out

$$\cancel{3^{\log_3}}(x^2-16) = 3^2$$

$x^2 - 16 = 9$ add 16 to both sides

$x^2 = 25$

Step 3: Square root both sides to solve for x

$$\sqrt{x^2} = \pm\sqrt{25} \quad \Rightarrow \quad x = \pm 5 \quad \Rightarrow \quad x = -5,\ x = 5$$

Step 4: Plug x values into function to determine if any logs are negative or zero

$x = -5$ gets negative values inside the log, which violates the domain, so the only solution is $x = 5$

Solve Exponential Equations

Example: Solve $3^{2x+1} = 4^x$

Step 1: Get x out of the exponent by introducing log or ln to both sides

$$\ln(3^{2x+1}) = \ln(4^x)$$

$$(2x+1)\ln 3 = x \ln 4$$

Step 2: Simplify to solve for x

distribute $2x\ln 3 + \ln 3 = x\ln 4$ subtract to get terms with x on the same side

$-2x\ln 3 \qquad -2x\ln 3$

$\ln 3 = x\ln 4 - 2x\ln 3$ factor out x

$\ln 3 = x(\ln 4 - 2\ln 3)$

$$\frac{\ln 3}{\ln 4 - 2\ln 3} = \frac{x\cancel{(\ln 4 - 2\ln 3)}}{\cancel{\ln 4 - 2\ln 3}}$$ get x by itself by dividing both sides

$$x = \frac{\ln 3}{\ln 4 - 2\ln 3} = -1.35$$