Factoring

Steps of factoring polynomials:

1) Look for greatest common factor (GCF)
2) Count number of terms
3) Determine method of factoring

Greatest Common Factor

Example 1: Factor $2x^4 + 6x^6$

Both terms have a $2x^4$ in common.

$$= 2x^4(1 + 3x^2)$$

Example 2: Factor $x(x + 19) + 11(x + 19)$

Both terms have $(x + 19)$ in common.

$$= (x + 19)(x + 11)$$

(4 terms) Factor by grouping

Example: Factor $2x^2 + 6x^3 + 5x^7 + 15x^8$

Step 1: Group terms into 2 groups of 2

$$\underbrace{2x^2 + 6x^3}\ + \underbrace{5x^7 + 15x^8}$$

Step 2: Factor GCF from each group

$$= 2x^2(1 + 3x) + 5x^7(1 + 3x)$$

(the binomials should be the same)

Step 3: Factor out the GCF (the binomial $(1 + 3x)$)

$$= (1 + 3x)(2x^2 + 5x^7)$$

(3 terms) Factor Trinomials with leading term coefficient of 1 In the form $x^2 + bx + c$

Example: Factor $x^2 - 16x + 55$

Step 1: Look for numbers that multiply to get 55

and add or subtract to get 16.

$11 \cdot 5 = 55$ and $11 + 5 = 16$

Step 2: Right as product of 2 binomials, with x^2 split into x and x

$(x \quad 11)(x \quad 5)$

Step 3: Determine signs

To multiply to get a positive 55, both need to be positive or both negative

To add to get a negative 16, both should be negative: $-11 + -5 = -16$

$= (x - 11)(x - 5)$

(3 terms) Factor Trinomial when leading term coefficient is not 1 (AC method)
In the form $ax^2 + bx + c$

Example: Factor $5x^2 + 31x + 6$

Step 1: Multiply first and last terms (a) and (c)

$5 \cdot 6 = 30$

Step 2: Find numbers that multiplied equal 30
and added or subtracted equal 31 (middle term)

30	
$5 \cdot 6$	no
$2 \cdot 15$	no
$30 \cdot 1$	yes $\quad 30 \cdot 1 = 30$ ✓ and $30 + 1 = 31$ ✓

Step 3: Create 4 term polynomial, replacing middle term

$5x^2 + 31x + 6$

$5x^2 + \_\_ + \_\_ + 6$

$5x^2 + 30x + 1x + 6$

Step 4: Factor by grouping

$5x^2 + 30x + 1x + 6$

$= 5x(x + 6) + 1(x + 6)$

$= (x + 6)(5x + 1)$

(2 terms) Difference of Squares
$a^2 - b^2 = (a - b)(a + b)$

Example 1: Factor $36x^2 - 49y^2$

$= (6x - 7y)(6x + 7y)$

Example 2: Factor $36x^2 + 49y^2$

Cannot factor (PRIME), sum of squares is not factorable

Example 3: Factor $6x^2 - 54$

Step 1: Factor out the Greatest Common Factor

$= 6(x^2 - 9)$

Step 2: Factor as difference of squares

$= 6(x - 3)(x + 3)$

(2 terms) Difference of Cubes and Sum of Cubes
Difference: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Sum: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Example: Factor $8x^3 + 125$

$\sqrt[3]{8x^3} = 2x$ and $\sqrt[3]{125} = 5$ so $a = 2x$ and $b = 5$

$(a + b)(a^2 - ab + b^2)$

$= (2x + 5)(4x^2 - 10x + 25)$

Solve Quadratic Equations by Factoring

Example 1: Solve $7x^2 = 6x$

Step 1: Move all terms to one side equal to zero

$$7x^2 - 6x = 0$$

Step 2: Factor out the Greatest Common Factor

$$x(7x - 6) = 0$$

Step 3: Set each factor to zero and solve each for x

$$x = 0 \qquad 7x - 6 = 0$$

$$x = 0 \qquad x = \frac{6}{7}$$

$$\left\{0, \frac{6}{7}\right\}$$

Example 2: Solve $x^2 - 3x = 18$

Step 1: Move all terms to one side equal to zero

$$x^2 - 3x - 18 = 0$$

Step 2: Factor the trinomial

$$(x - 6)(x + 3) = 0$$

Step 3: Set each factor to zero and solve each for x

$$x - 6 = 0 \qquad x + 3 = 0$$

$$x = 6 \qquad x = -3$$

$$\{6, -3\}$$