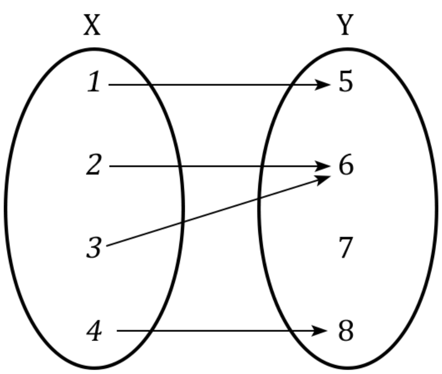
Functions

Input	Output
$x,\ t,\ \theta, \ldots$ First coordinate Independent variable Domain is the set of all inputs	$y,\ f(x),\ g(t),\ h(\theta), \ldots$ Second Coordinate Dependent variable Range is the set of all outputs

A function relates a set of inputs with a set of outputs, where each input is related to *exactly one* output.

Example 1:

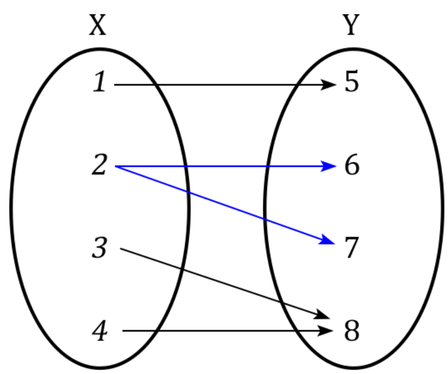
A function:



(every input, x, has only one output, y)

Example 2:

NOT a function:



(the x value 2 has more than one output, 6 and 7)

Example 3: A function: $\{(2,3)\ (3,1)\ (4,5)\}$

This is a function, since in each ordered pair (x, y), each x corresponds to only one y.

Example 4: NOT a function: $\{(2,3)\ (2,1)\ (4,5)\}$

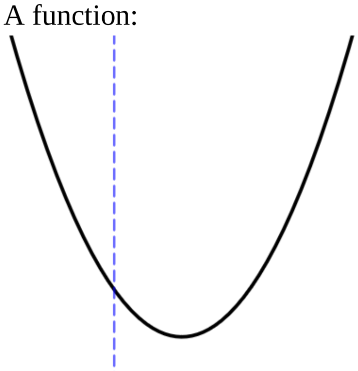
This is NOT a function, since the x value 2 corresponds to 3 *and* 1, so has more than one output y.

Vertical Line Test

– if a vertical line passes through a graph more than once, the graph is NOT a function.

Example 1:

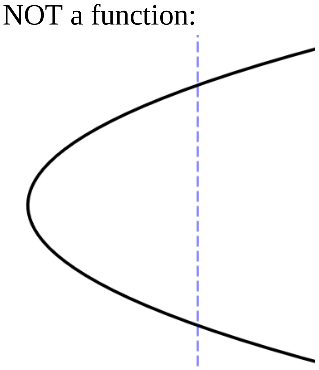
A function:



(Any vertical line only passes through once)

Example 2:

NOT a function:



(A vertical line passes through the graph more than once)

Functions

If equation has y^2 (or y to any even power), or $|y|$, it is NOT a function.

Examples of equations that are functions:	Examples of equations that are NOT functions:
$x + 2y = 4$	$x + 3y^2 = 1$
$y = \frac{1}{x}$	$x = \frac{1}{y^2}$
$x^2 + y^3 = 5$	$y^4 - 5y = 8x$
$y = \|x\|$	$x = \|y\|$

Evaluating Functions

Example 1: For the function $f(x) = x^2 - 3x + 4$
Find: $f(-1)$

Step 1: Plug -1 in for x

$$f(-1) = (-1)^2 - 3(-1) + 4 \quad = 1 + 3 + 4$$

$$f(-1) = 8$$

Example 2: For the function $h(t) = 2t^2 - 8t + 3$
Find: $h(4)$

Step 1: Plug 4 in for t

$$h(4) = 2(4)^2 - 8(4) + 3 \quad = 2(16) - 32 + 3 \quad = 32 - 32 + 3$$

$$h(4) = 3$$

Example 3: For the function $g(x) = 3x^2 - 5x - 2$
Find: $g(a)$

Step 1: Plug a in for x

$$g(a) = 3(a)^2 - 5(a) - 2$$

$$g(a) = 3a^2 - 5a - 2$$

Example 4: For the function $f(x) = 4x + 5$
Find: $f(x + h)$

Step 1: Plug in $(x + h)$ for x

$$f(x + h) = 4(x + h) + 5$$

Distribute $= 4x + 4h + 5$

$$f(x + h) = 4x + 4h + 5$$

Evaluating Functions (difference quotient)

Example 5: For the function $f(x) = 3x^2 - 2x + 1$

Find: $\dfrac{f(x+h) - f(x)}{h}$ This formula is often called the difference quotient.

Step 1: Find $f(x+h)$

Plug in $(x+h)$ for x in the function $f(x) = 3x^2 - 2x + 1$

$$f(x+h) = 3(x+h)^2 - 2(x+h) + 1$$

Simplify $= 3(x+h)(x+h) - 2(x+h) + 1$

$$= 3(x^2 + 2xh + h^2) - 2x - 2h + 1$$

$$f(x+h) = 3x^2 + 6xh + 3h^2 - 2x - 2h + 1$$

Step 2: Plug in $f(x+h)$ and $f(x)$ into $\dfrac{f(x+h) - f(x)}{h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{(3x^2 + 6xh + 3h^2 - 2x - 2h + 1) - (3x^2 - 2x + 1)}{h}$$

Step 3: Combine like terms and simplify

$$= \frac{(\cancel{3x^2} + 6xh + 3h^2 - \cancel{2x} - 2h + \cancel{1}) - (\cancel{3x^2} - \cancel{2x} + \cancel{1})}{h}$$

$$= \frac{6xh + 3h^2 - 2h}{h}$$

Factor $= \dfrac{\cancel{h}(6x + 3h - 2)}{\cancel{h}}$

$$= 6x + 3h - 2$$

Domain and Range

Domain: the set of all inputs (what x can be).	Range: the set of all outputs (what y can be).

The domain will be all real numbers $(-\infty, \infty)$ unless the function has:

- x in the denominator
- x in a square root (or any even root)
- x in a logarithm

Example 1: Find the domain of the function $f(x) = x^2 + 3$

The domain is all real numbers, $(-\infty, \infty)$
any real number plugged in for x will get a real number as an output.

Example 2: Find the domain of the function $g(x) = \dfrac{x+9}{x+7}$

Step 1: Note that an expression is undefined when the denominator equals zero.
x is in the denominator, so set the denominator to zero and solve for x

$x + 7 = 0 \;\Rightarrow\; x = -7$

So the function will be undefined when $x = -7$

Therefore, the domain is all real numbers except $x = -7$

The domain in interval notation is: $(-\infty, -7) \cup (-7, \infty)$

Example 3: Find the domain of the function $h(x) = \sqrt{3 - x}$

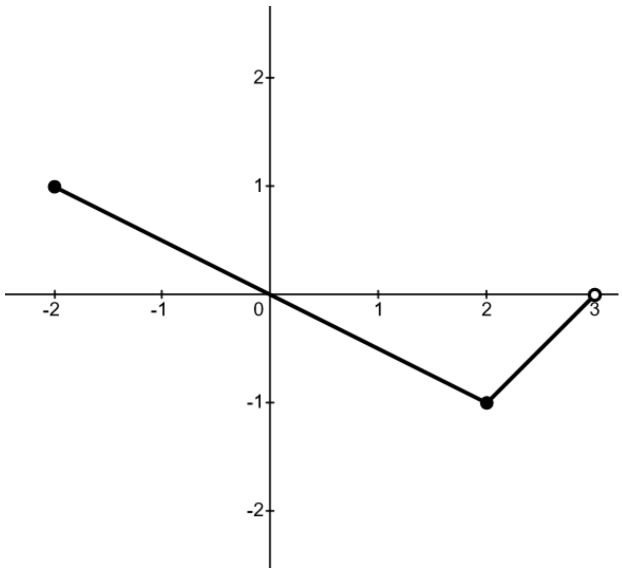
Step 1: Note that an expression is not a real number when the taking the square root of a negative number.
Since x is inside a square root, set the inside of the root ≥ 0 and solve for x.

$3 - x \geq 0 \;\Rightarrow\; -x \geq -3$ divide both sides by -1, flipping the inequality $x \leq 3$

So the domain is $x \leq 3$

The domain in interval notation is: $(-\infty, 3]$

Example 4: Find the domain and range of the graph



Step 1: Look at the far left and far right for domain.
Step 2: Look at the lowest and highest for the range.

Domain: $[-2, 3)$
Range: $[-1, 1]$

Note:

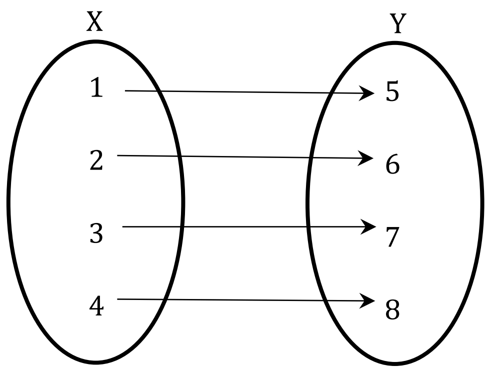
Use brackets [ ] for closed circle
Use parenthesis ( ) for open circle

One-to-One Functions

A function is one-to-one when each output(y) is related to *exactly one* input(x).

Example 1:

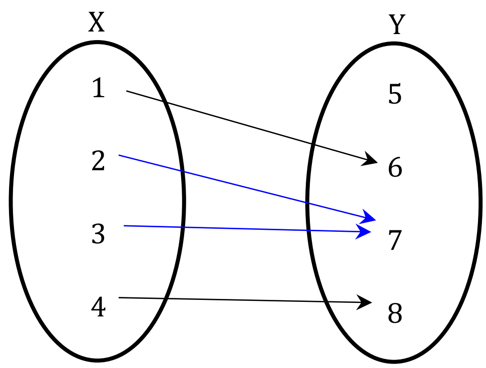
One-to-One Function:



(every output, y, has only one input, x)

Example 2:

NOT a One-to-One Function:



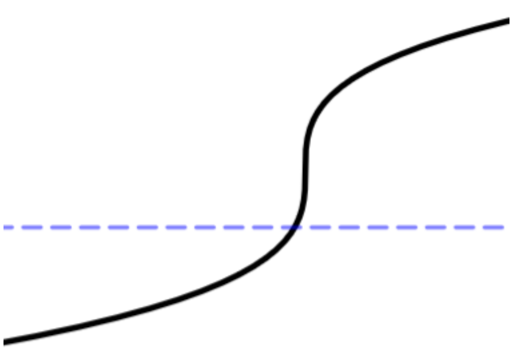
(the y value 7 has more than one input, 2 and 3)

Horizontal Line Test

– if a horizontal line passes through a graph more than once, the graph is NOT a one-to-one function.

Example 1:

A one-to-one function:



Example 2:

NOT a one-to-one function: