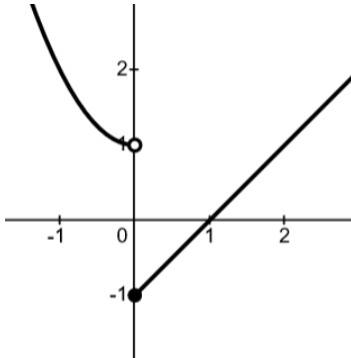# Piecewise Functions

Piecewise function – a function defined by two or more equations, each with different domains.

Example: A piecewise function with 2 pieces:

$$f(x) = \begin{cases} x^2 + 1, & \text{if } x < 0 \\ x - 1, & \text{if } x \geq 0 \end{cases}$$

In words, this is saying,

the function is $x^2 + 1$ when x is less than zero.

the function is $x - 1$ when x is greater than or equal to zero.

Note: closed circle ● is used for $\leq$ and $\geq$

open circle ○ is used for $<$ and $>$

Example: Find the following: For the function:

a) $f(-2)$

b) $f(-1)$

$$f(x) = \begin{cases} 2x + 4 & \text{if } x \leq -1 \\ 5 & \text{if } -1 < x < 2 \\ x^2 + 1 & \text{if } x \geq 2 \end{cases}$$

c) $f(0)$

d) $f(5)$

a) -2 is less than -1, so plug -2 into $2x + 4$

$$f(-2) = 2(-2) + 4 \ = -4 + 4 \ = 0$$
$$f(-2) = 0$$

b) -1 is equal to -1, so plug -1 into $2x + 4$

$$f(-1) = 2(-1) + 4 \ = -2 + 4 \ = 2$$
$$f(-1) = 2$$

c) 0 is between -1 and 2, so plug 0 into 5

There is no x variable to plug in, so the function value remains 5
$$f(0) = 5$$

d) 5 is greater than 2, so plug 5 into $x^2 + 1$

$$f(5) = (5)^2 + 1 \ = 25 + 1 \ = 26$$
$$f(5) = 26$$

Graphing Piecewise Functions

Example: Graph the function:

$$f(x) = \begin{cases} -x + 3 & \text{if } x \leq -1 \\ x^2 + 1 & \text{if } -1 < x < 2 \\ 5 & \text{if } x \geq 2 \end{cases}$$

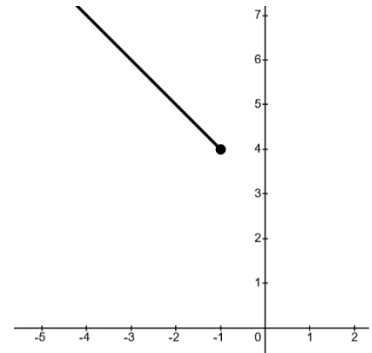Step 1: For $-x + 3$ if $x \leq -1$, plug in -1 for x, and any number below -1 for x.

x	$f(x)$
-1	4
-2	5
-3	6

$f(-1) = -(-1) + 3 = 4$

$f(-2) = -(-2) + 3 = 5$

$f(-3) = -(-3) + 3 = 6$

$(-1,4)$ is an endpoint

Step 2: For $x^2 + 1$ if $-1 < x < 2$, plug in -1 and 2 for x, and any number between -1 and 2 for x.

x	$f(x)$
-1	2
2	5
0	1

$f(-1) = (-1)^2 + 1 = 2$
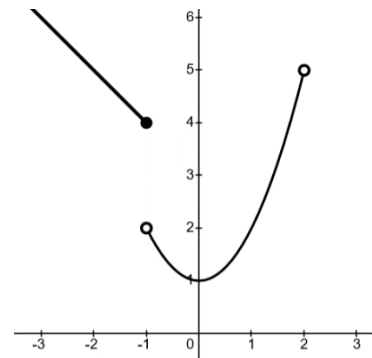
$f(2) = (2)^2 + 1 = 5$

$f(0) = (0)^2 + 1 = 1$

$(-1,2)$ is an endpoint

$(2,5)$ is an endpoint

Step 3: For 5 if $x \geq 2$, plug in 2 for x, and any number above 2 for x.
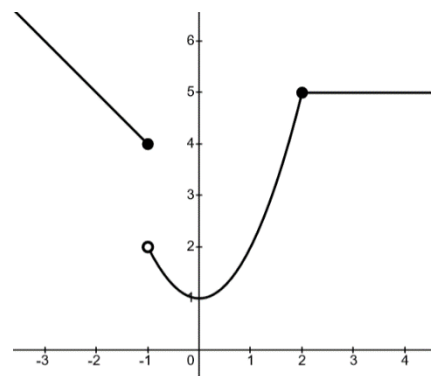
x	$f(x)$
2	5
3	5
4	5

$f(2) = 5$

$f(3) = 5$

$f(4) = 5$

$(2,5)$ is an endpoint

Graph of all pieces of the function.