Properties of Exponents

	Property	**Example**
For any positive integer n, where a is the base and n is the exponent:	$a^n = \underbrace{a \cdot a \cdot a \ldots a}_{n \text{ factors}}$	$2^3 = 2 \cdot 2 \cdot 2 = 8$
Exponent of 1:	$a^1 = a$	$5^1 = 5$
Exponent of zero: (when $a \neq 0$)	$a^0 = 1$	$6^0 = 1$
Exponent of -1: (when $a \neq 0$)	$a^{-1} = \frac{1}{a^1} = \frac{1}{a}$	$9^{-1} = \frac{1}{9}$
	$\frac{1}{a^{-1}} = a^1 = a$	$\frac{1}{3^{-1}} = 3^1 = 3$
Negative exponents: (when $a \neq 0$)	$a^{-n} = \frac{1}{a^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
	$\frac{1}{a^{-n}} = a^n$	$\frac{1}{2^{-3}} = 2^3 = 8$
	$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$	$\frac{2^{-4}}{5^{-3}} = \frac{5^3}{2^4} = \frac{125}{16}$
Product Rule:	$a^m \cdot a^n = a^{m+n}$	$4^2 \cdot 4^3 = 4^{2+3} = 4^5 = 1024$
Raising a product to a power:	$(ab)^n = a^n \cdot b^n$	$(3x)^4 = 3^4 \cdot x^4 = 81x^4$
Quotient Rule: (when $a \neq 0$)	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{4^7}{4^3} = 4^{7-3} = 4^4 = 256$
		$\frac{4^3}{4^7} = 4^{3-7} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$ or $\frac{4^3}{4^7} = \frac{1}{4^{7-3}} = \frac{1}{4^4} = \frac{1}{256}$
Raising a quotient to a power: (when $b \neq 0$)	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$
Power Rule:	$(a^m)^n = a^{mn}$	$(5^3)^4 = 5^{3 \cdot 4} = 5^{12}$

Example 1: Simplify $(4xy^3)^2$

Step 1: Distribute the exponent outside the parenthesis

$$(4xy^3)^2 \quad = 4^2x^2y^6 \quad = 16x^2y^6$$

Example 2: Simplify $\dfrac{32x^{-5}y^{-1}z^2}{4x^{-2}y^8}$

Step 1: Flip the negative exponents

$$\frac{32x^{-5}y^{-1}z^2}{4x^{-2}y^8} \quad = \frac{32x^2z^2}{4x^5yy^8}$$

Step 2: Combine like terms and reduce fractions

$$\frac{32x^2z^2}{4x^5yy^8} \quad = \frac{8z^2}{x^{(5-2)}y^{(1+8)}} \quad = \frac{8z^2}{x^3y^9}$$

Properties of Rational (Fractional) Exponents

	Property:	**Example:**
Power of $1/2$ (square root)	$a^{1/2} = \sqrt{a}$	$9^{1/2} = \sqrt{9} = 3$
Power of $1/3$ (cube root)	$a^{1/3} = \sqrt[3]{a}$	$8^{1/3} = \sqrt[3]{8} = 2$
Power of $1/n$ (nth root)	$a^{1/n} = \sqrt[n]{a}$	$5^{1/7} = \sqrt[7]{5}$
Power of m/n	$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$	$7^{3/5} = \sqrt[5]{7^3}$ or $\left(\sqrt[5]{7}\right)^3$
Negative Exponent	$a^{-m/n} = \dfrac{1}{a^{m/n}}$	$7^{-2/5} = \dfrac{1}{7^{2/5}} = \dfrac{1}{\sqrt[5]{7^2}}$

Properties of Radicals		
	Property:	**Example:**
If n is odd:	$\sqrt[n]{a^n} = a$	$\sqrt[3]{(-5)^3} = -5$
If n is even:	$\sqrt[n]{a^n} = \lvert a \rvert$	$\sqrt[4]{(-5)^4} = 5$
	$\sqrt[2]{a} = \sqrt{a}$	
	$\sqrt{a^2} = \lvert a \rvert$	$\sqrt{(-5)^2} = 5$
	$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$	$\sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$
	$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$	$\sqrt{3} \cdot \sqrt{5} = \sqrt{15}$
	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$\sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{5}{3}$
	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt[4]{\sqrt[3]{a}} = \sqrt[12]{a}$