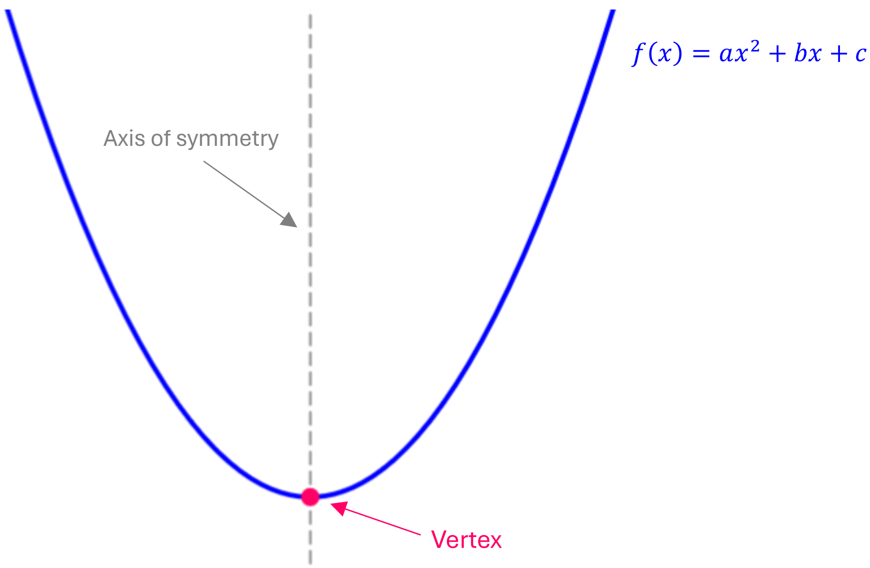
Quadratic Functions

A quadratic function is a function where the largest exponent of the variable is two.

Examples of quadratic functions:	
$f(x) = x^2$	$2y + 5x^2 = -8x + 4y$
$y = 3x^2 - 7x + 6$	$C(t) = -0.7t^2 - 4x + 9$
$g(x) = (x-3)(x+4)$	$f(x) = (x-3)^2 + 5$

The graph of a quadratic function is a parabola:



Standard Form of a Quadratic: $f(x) = ax^2 + bx + c$

where a, b and c area real numbers and $a \neq 0$.

Example: $f(x) = 3x^2 - 24x + 49$

Then $a = 3,\ \ b = -24,\ \ c = 49$

Vertex Form of a Quadratic: $f(x) = a(x-h)^2 + k$

where the vertex is the point (h, k).

Example: $f(x) = 3(x-4)^2 + 1$

In this form we can see the vertex is $(4, 1)$.

Vertex

The vertex of a quadratic equation is the **maximum** or **minimum** point of the function.

Vertex Formula:	The x coordinate of the vertex is $x = \frac{-b}{2a}$ (This is also the formula for axis of symmetry) Then plug in the x coordinate into the function to find the y coordinate. The vertex can also be shown as the point (h, k), where $h = \frac{-b}{2a}$ and $k = f(h)$

Example: Find the vertex and x and y-intercepts and sketch the parabola

$$f(x) = -x^2 - 6x + 7$$

Step 1: Note the function is in the form $ax^2 + bx + c$, and $a = -1$, $b = -6$, $c = 7$

Step 2: Plug into the formula to find the x coordinate of the vertex.

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = \frac{6}{-2} = -3$$

$$x = -3$$

Step 3: Plug x coordinate into function to get the y coordinate of the vertex.

$$f(-3) = -(-3)^2 - 6(-3) + 7 = 16$$

$$y = 16$$

So the vertex is the point $(-3, 16)$.

Step 4: Note the negative on the leading term of the functions means the graph is going down.
Plot the vertex point and find x and y-intercepts to graph the function.

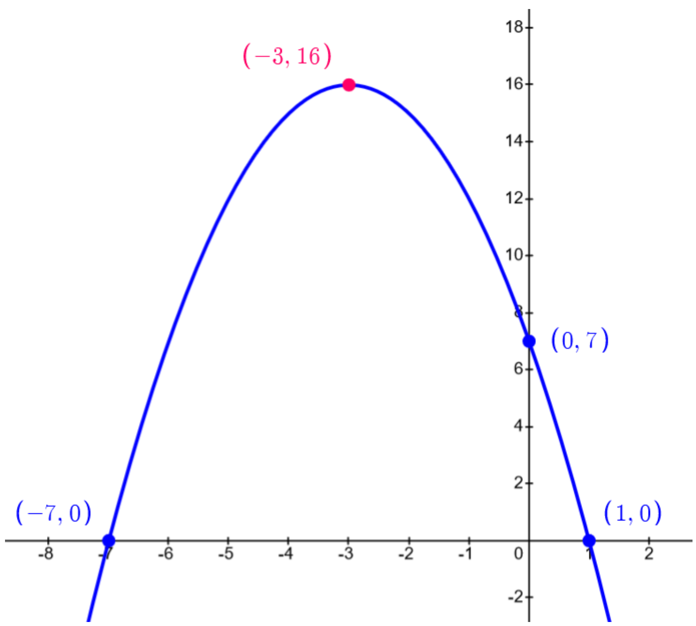
To find x-intercepts, set function to zero and solve for x, by factoring *or* using the quadratic formula.

$-x^2 - 6x + 7 = 0$
$-(x^2 + 6x - 7) = 0$
$-(x - 1)(x + 7) = 0$
$x - 1 = 0 \quad x + 7 = 0$
$x = 1, x = -7$
So x-intercepts are $(1,0)$ and $(-7,0)$

To find the y-intercept, plug zero in for x and solve for y.

$f(0) = -(0)^2 - 6(0) + 7 = 7$

So the y-intercept is $(0, 7)$.



Solve Quadratic Equations by Factoring

Example 1: Solve $7x^2 = 6x$

Step 1: Move all terms to one side equal to zero

$$7x^2 - 6x = 0$$

Step 2: Factor out the Greatest Common Factor

$$x(7x - 6) = 0$$

Step 3: Set each factor to zero and solve each for x

$$x = 0 \qquad 7x - 6 = 0$$

$$x = 0 \qquad x = \frac{6}{7}$$

$$\left\{0, \frac{6}{7}\right\}$$

Example 2: Solve $x^2 - 3x = 18$

Step 1: Move all terms to one side equal to zero

$$x^2 - 3x - 18 = 0$$

Step 2: Factor the trinomial

$$(x - 6)(x + 3) = 0$$

Step 3: Set each factor to zero and solve each for x

$$x - 6 = 0 \qquad x + 3 = 0$$

$$x = 6 \qquad x = -3$$

$$\{-3, 6\}$$

Example 3: Solve $x^2 = 9$

Step 1: Move all terms to one side equal to zero

$$x^2 - 9 = 0$$

Step 2: Factor using the difference of squares $a^2 - b^2 = (a - b)(a + b)$

$$x^2 - 9 = 0$$

$$(x - 3)(x + 3) = 0$$

$$x - 3 = 0 \qquad x + 3 = 0$$

$$x = 3 \qquad x = -3$$

$$\{-3, 3\}$$

Note: This can also be solved using the square root method.

Solve Quadratic Equations by Square Root Method

A square root and a square cancel each other out since they are inverses of each other.

$$\sqrt{x^2} = x$$

Example 1: Solve $x^2 = 9$ using square root method.

Step 1: Take the square root of both sides of the equation to cancel the square.

$x^2 = 9$

$\sqrt{x^2} = \pm\sqrt{9}$ Note, include $\pm$ on the number when square rooting.

Step 2: Simplify

$x = \pm 3$ So $x = -3$ and $x = 3$

$\{-3, 3\}$

Example 2: Solve $3(x-4)^2 - 40 = 5$

Step 1: Isolate the square term $(x-4)^2$ by first adding 40 to both sides.

$3(x-4)^2 = 45$

Then divide both sides by 3

$(x-4)^2 = 15$

Step 2: Take the square root of both sides to cancel the square.

$\sqrt{(x-4)^2} = \pm\sqrt{15}$

Step 3: Simplify

$x - 4 = \pm\sqrt{15}$

Add 4 to both sides

$x = 4 \pm \sqrt{15}$ So $x = 4 - \sqrt{15}$ and $x = 4 + \sqrt{15}$

$\{\ 4 - \sqrt{15},\ \ 4 + \sqrt{15}\ \}$

Solve Quadratic Equations Using the Quadratic Formula

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 1: Solve $x^2 - 3x = 18$ using the quadratic formula.

Step 1: Move all terms to one side equal to zero

$$x^2 - 3x - 18 = 0$$

Step 2: Identify a, b and c and plug into the quadratic formula

$$a = 1, \quad b = -3, \quad c = -18$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-18)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{81}}{2} \quad \Rightarrow \quad x = \frac{3 \pm 9}{2} \quad \Rightarrow \quad x = \frac{3-9}{2}, \ x = \frac{3+9}{2}$$

$$x = -3 \qquad x = 6 \qquad \{-3, 6\}$$

Note: This can also be solved by factoring.

Example 2: Solve $3x^2 = 4x + 6$ using the quadratic formula.

Step 1: Move all terms to one side equal to zero

$$3x^2 - 4x - 6 = 0$$

Step 2: Identify a, b and c and plug into the quadratic formula

$$a = 3, \quad b = -4, \quad c = -6$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-6)}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{88}}{6} \quad \Rightarrow \quad x = \frac{4 \pm \sqrt{4 \cdot 22}}{6} \quad \Rightarrow \quad x = \frac{4 \pm 2\sqrt{22}}{6} \quad \Rightarrow \quad x = \frac{2 \pm \sqrt{22}}{3}$$

$$x = \frac{2 - \sqrt{22}}{3}, \quad x = \frac{2 + \sqrt{22}}{3} \qquad \left\{ \frac{2 - \sqrt{22}}{3}, \frac{2 + \sqrt{22}}{3} \right\}$$