Slope of a Line

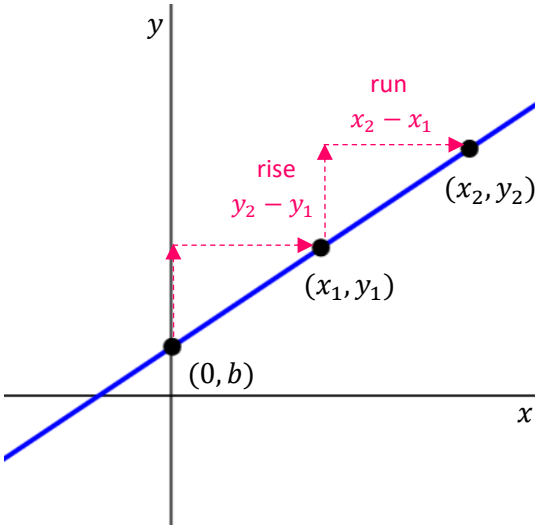
The slope is a measure of the steepness and direction(negative or positive) of a line.

Slope is often shown as the variable m

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{change\ in\ y}{change\ in\ x} = \frac{rise}{run}$

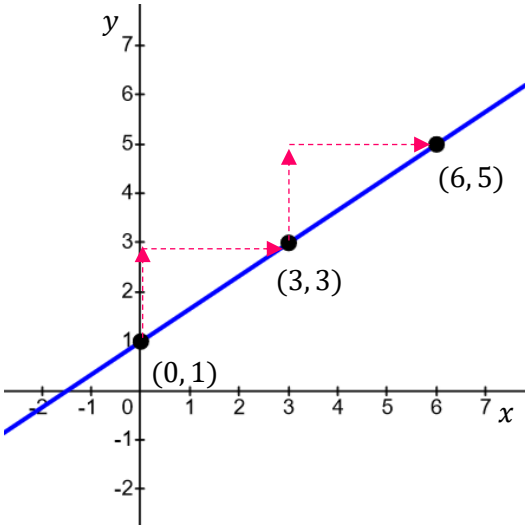
Equation of a Line (Linear Equation)

Slope-Intercept Form: $y = mx + b$,
where $m =$ slope and $b = y$-intercept



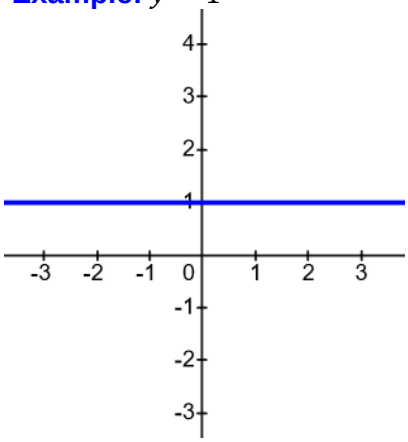
Example: $y = \frac{2}{3}x + 1$

$m = \frac{2}{3}$ and $b = 1$



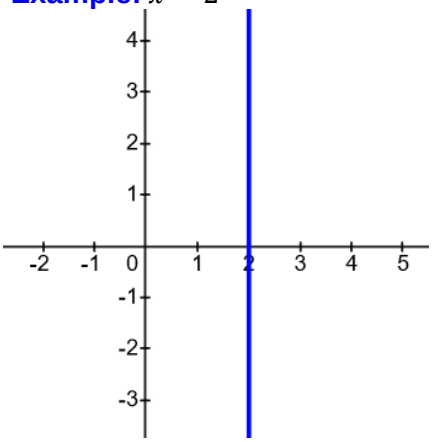
A Horizontal Line has the equation $y = b$
and a slope of zero.

Example: $y = 1$



A Vertical Line has the equation $x = a$
and undefined slope.

Example: $x = 2$



Graphing using the Slope and y-intercept

To graph a linear equation, using slope and y-intercept:

1) Solve the equation for y to put it in $y = mx + b$ form if necessary.
2) Identify the slope and y-intercept.
2) Plot the y-intercept.
3) Starting at y-intercept point, move the rise and run to get another point.
4) You can get more points from moving the rise and run from any existing point.

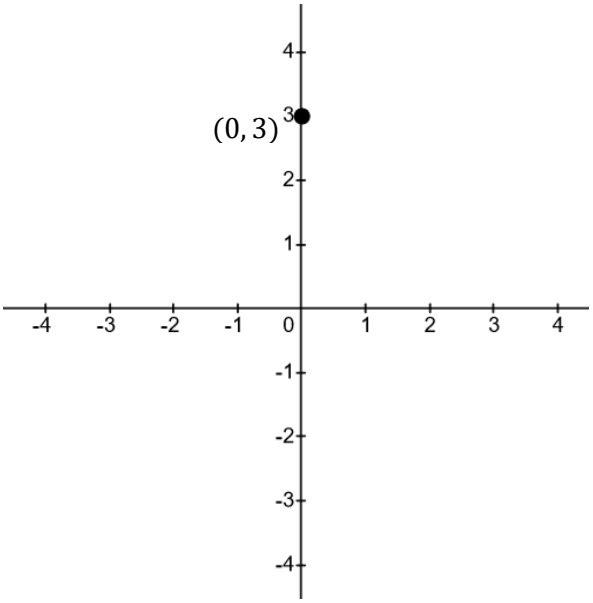
Example: Graph the equation $y = -x + 3$

Step 1: The equation is already in $y = mx + b$ form, so determine the slope and y-intercept

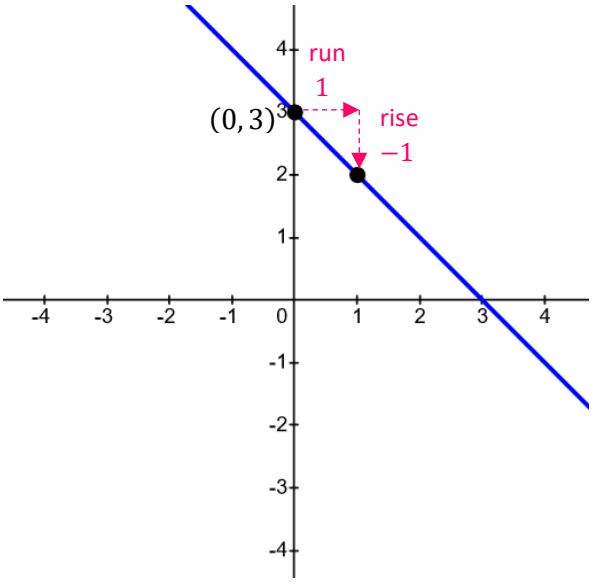
The slope is $m = -1$ To see as rise and run, put the number over 1. So $m = \frac{-1}{1} = \frac{rise}{run}$

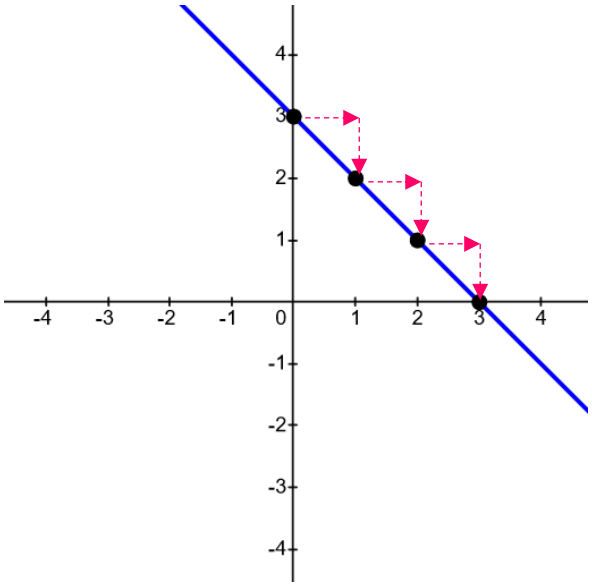
The y-intercept is 3, and as a point $(0, 3)$

Step 2: Plot the y-intercept



Step 3: From the y-intercept, move the rise and run of the slope to get another point, and connect them.
(Move the rise and run from the new point to get more points)





Forms and Formulas

m = slope, b = y-intercept

- Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Parallel lines have the same slope
  Perpendicular lines have the opposite slope (negative reciprocal) $\dfrac{a}{b} \rightarrow -\dfrac{b}{a}$
- Slope-Intercept Form: $y = mx + b$ To find equation, if you have a y-intercept and slope, plug in to slope-intercept form.
- Point-Slope Form: $y - y_1 = m(x - x_1)$ To find equation, if you have a random point and a slope, plug in to point-slope form.
- Standard Form: $Ax + By = C$

Writing Equations

Example 1: Write an equation in slope-intercept form
that passes through the point $(2,3)$
and is *parallel* to the equation $2x + y = 4$

Step 1: Find the slope of the given equation by writing the equation in slope-intercept form

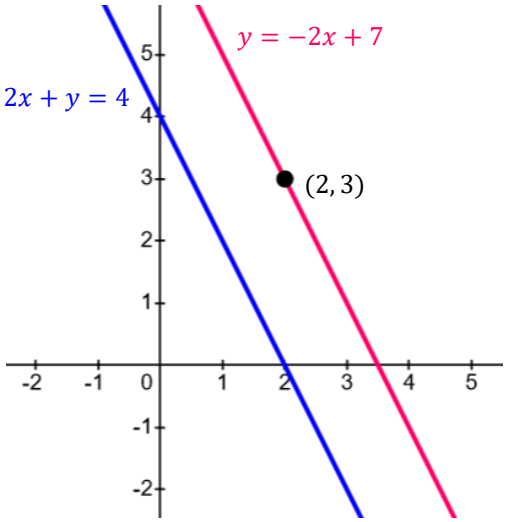
$2x + y = 4 \quad \Rightarrow \quad y = -2x + 4$ So we can see the slope of the given equation is $m = -2$
Since we want an equation that is parallel, we use the same slope.

Step 2: Plug in the given point and the slope into the point-slope formula and solve for y

$y - y_1 = m(x - x_1)$

$y - 3 = -2(x - 2) \quad \Rightarrow \quad y - 3 = -2x + 4 \quad \Rightarrow \quad y = -2x + 7$

Graph to confirm:



(Note that parallel lines *do not* intersect)

Example 2: Write an equation in slope-intercept form
that passes through the point $(2, 3)$
and is *perpendicular* to the equation $2x + y = 4$

Step 1: Find the slope of the given equation by writing the equation in slope-intercept form

$2x + y = 4 \quad \Rightarrow \quad y = -2x + 4$ So we can see the slope of the given equation is $m = -2$

Since we want an equation that is perpendicular, find the negative reciprocal of -2

$-2 = \frac{-2}{1}$ Flip and change sign to get perpendicular slope $-\frac{2}{1} \rightarrow \frac{1}{2}$

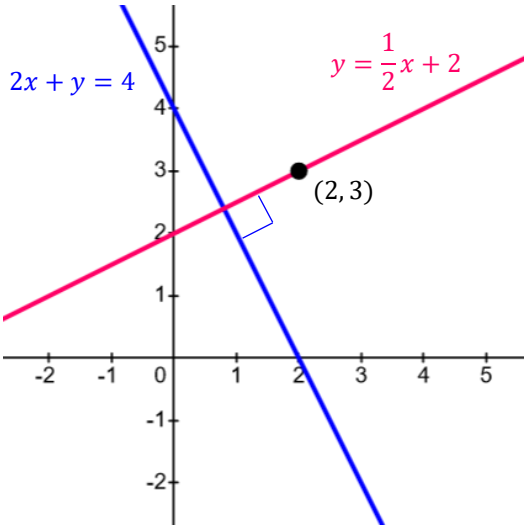
So the perpendicular slope is $m = \frac{1}{2}$

Step 2: Plug in the given point and perpendicular slope into the point-slope formula and solve for y

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{2}(x - 2) \quad \Rightarrow \quad y - 3 = \frac{1}{2}x - 1 \quad \Rightarrow \quad y = \frac{1}{2}x + 2$

Graph to confirm:



(Note that perpendicular lines intersect at a right angle)