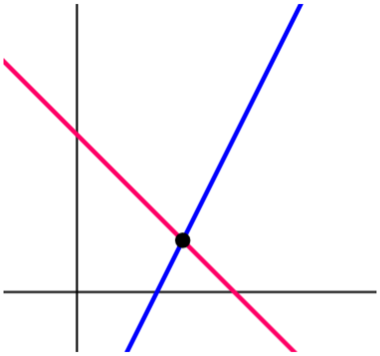
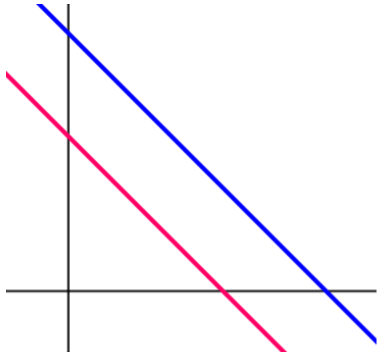
System of Equations

Types of solutions:

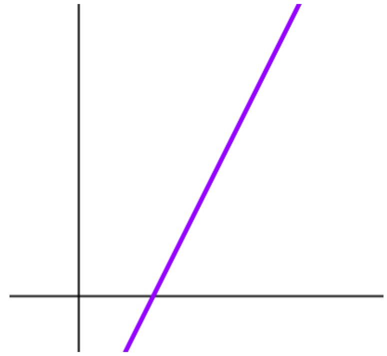
One Solution (intersecting lines)
(consistent, independent)



No Solution (parallel lines)
(inconsistent, independent)



Infinite Solutions (overlapping lines)
(consistent, dependent)



Note:
When solving, if variables cancel and you get a true expression such as $3 = 3$ or $0 = 0$, that is infinite solutions.
If variables cancel and you get an untrue expression such as $0 = 5$ or $6 = 4$, that is no solution.

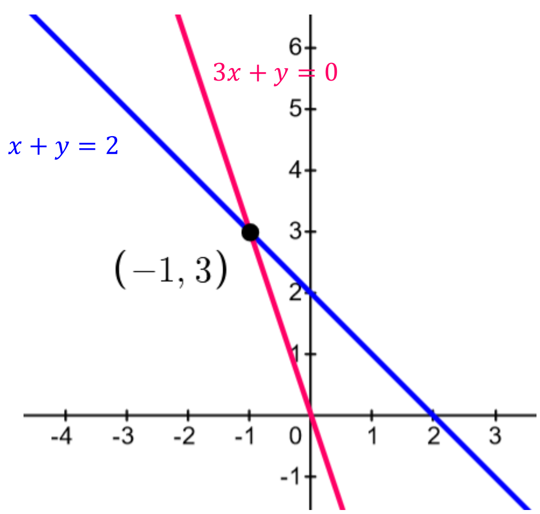
Solve by Graphing

Example: Solve the following system of equations graphically

$$\begin{cases} x + y = 2 \\ 3x + y = 0 \end{cases}$$

Step 1: Graph both equations by plotting points or using slope and y-intercept.

Observe where the lines intersect.



The solution is $(-1, 3)$.

Solve by Substitution Method

Example: Solve using the substitution method:

$$\begin{cases} 3x - y = 2 \\ 8x - 2y = 8 \end{cases}$$

Step 1: Solve either equation for x or y.

Let's solve the top equation for y.

$$3x - y = 2$$

$$-y = -3x + 2 \quad \Rightarrow \quad y = 3x - 2$$

Step 2: Plug in for y into the other equation.

$$8x - 2y = 8$$

$$8x - 2(3x - 2) = 8$$

Simplify to solve for x

$$8x - 6x + 4 = 8$$

$$2x + 4 = 8$$

$$x = 2$$

Step 3: Plug x in to one of the original equations to find y

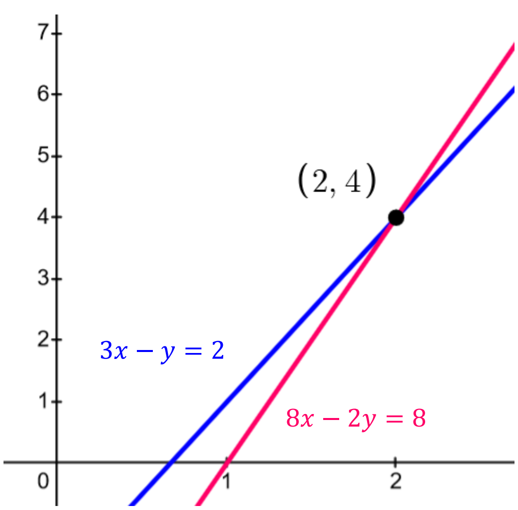
Let's plug into the equation from step 1: $y = 3x - 2$

$$y = 3(2) - 2 \quad = 6 - 2 \quad = 4$$

$$y = 4$$

So the solution is $(2, 4)$

Graph to confirm:



Solve by Elimination Method

Example: Solve using the elimination method:

$$\begin{cases} 7x - 6y = 8 \\ 2x + 5y = 9 \end{cases}$$

Step 1: Multiply to get x or y terms the same number with opposite signs.

$$\begin{array}{r} 2(7x - 6y = 8) \\ -7(2x + 5y = 9) \end{array} \longrightarrow \begin{array}{r} 14x - 12y = 16 \\ -14x - 35y = -63 \end{array}$$

Step 2: Add equations.

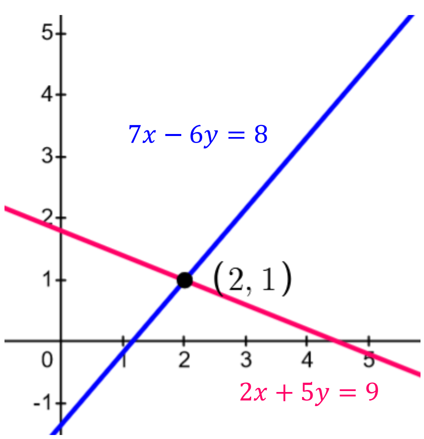
$$\begin{array}{r} 14x - 12y = 16 \\ -14x - 35y = -63 \\ \hline -47y = -47 \\ y = 1 \end{array}$$

Step 3: Plug in to one of the original equations to solve for the other variable.

$$2x + 5y = 9$$
$$2x + 5(1) = 9$$
$$2x + 5 = 9$$
$$2x = 4$$
$$x = 2$$

So the solution is $(2, 1)$

Graph to confirm:



Systems of Equations in Three Variables

General Steps to solve for three variables:
1) use two equations at a time
2) eliminate a variable to get two, two variable equations

Example: Solve the following system:
$$\begin{cases} x+y+z=2 \\ 6x-4y+5z=31 \\ 5x+2y+2z=13 \end{cases}$$

Step 1: Eliminate a variable in 2 equations.
Let's eliminate x using the 1st and 2nd equations.

$$\begin{cases} -6(x+y+z=2) \\ 6x-4y+5z=31 \end{cases} \qquad \begin{cases} -6x-6y-6z=-12 \\ 6x-4y+5z=31 \end{cases}$$
$$-10y-z=19$$

Step 2: Eliminate the same variable using the equation you didn't use yet (3rd eq.) and either of the other equations, in order to get two, two variable equations with the same variables.

$$\begin{cases} -5(x+y+z=2) \\ 5x+2y+2z=13 \end{cases} \qquad \begin{cases} -5x-5y-5z=-10 \\ 5x+2y+2z=13 \end{cases}$$
$$-3y-3z=3$$

Step 3: Bring the two, two variable equations together and solve using elimination or substitution.

$$\begin{cases} -10y-z=19 \\ -3y-3z=3 \end{cases}$$

Using elimination,

$$-3(-10y-z=19) \qquad 30y+3z=-57$$
$$-3y-3z=3 \qquad -3y-3z=3$$
$$27y=-54 \longrightarrow y=-2$$

Plugging in y to either two variable equation,

$$-3(-2)-3z=3 \quad \Rightarrow \quad 6-3z=3 \quad \Rightarrow \quad z=1$$

Step 4: Plug both variables into any of the three variable equations to find the third variable.

Using the first equation $x+y+z=2$

$$x+(-2)+(1)=2$$
$$x-1=2 \quad \Rightarrow \quad x=3$$

So the solution is $(3,-2,1)$