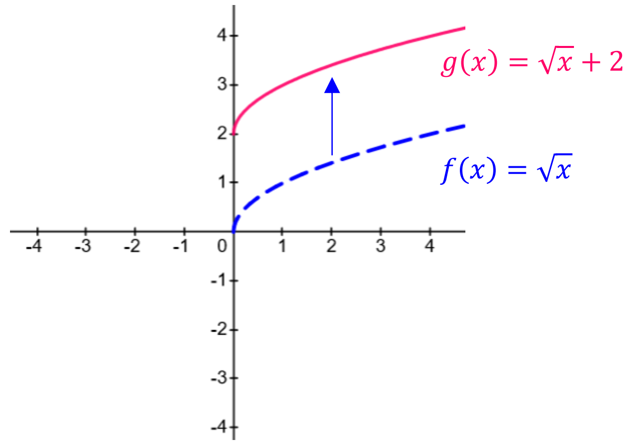
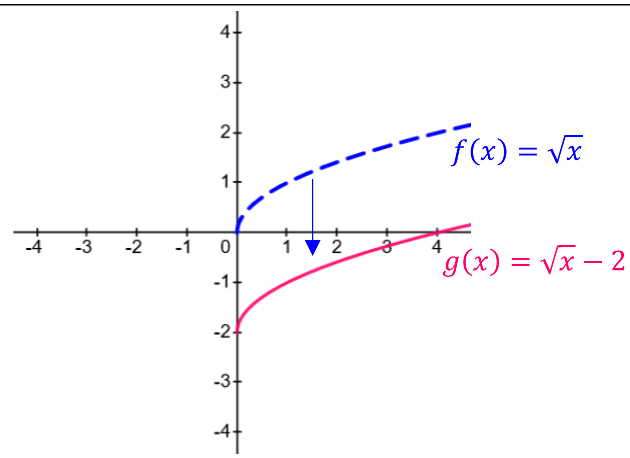
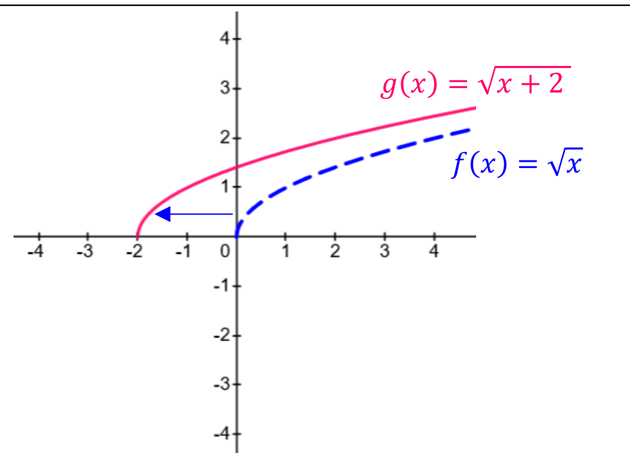
# Transformations of Functions

Transformations of functions are functions made by the shifting, reflecting, or scaling of another function.

**Example:** We can use an example function of $f(x) = \sqrt{x}$ to illustrate all the types of transformations.

| Transformation | Notation | Graph |
|---|---|---|
| **Vertical shift 2 units up:** | $g(x) = f(x) + 2$<br>$g(x) = \sqrt{x} + 2$ |  |
| **Vertical shift 2 units down:** | $g(x) = f(x) - 2$<br>$g(x) = \sqrt{x} - 2$ |  |
| **Horizontal shift 2 units left:** | $g(x) = f(x + 2)$<br>$g(x) = \sqrt{x + 2}$ |  |

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**Horizontal shift 2 units right:**

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$g(x) = f(x - 2)$

$g(x) = \sqrt{x - 2}$

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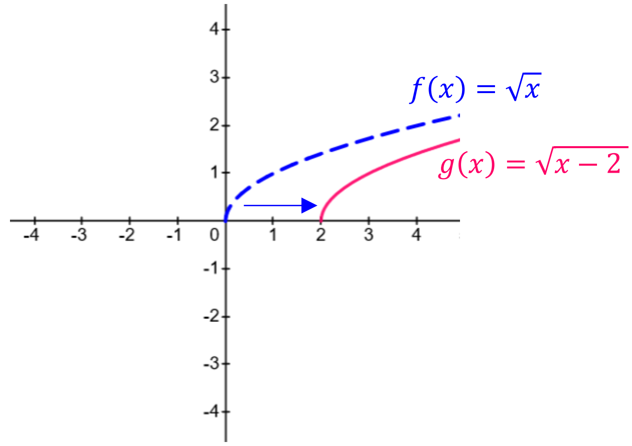


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**Reflection over the $x$-axis:**
(folding the graph at the $x$-axis)

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$g(x) = -f(x)$

$g(x) = -\sqrt{x}$

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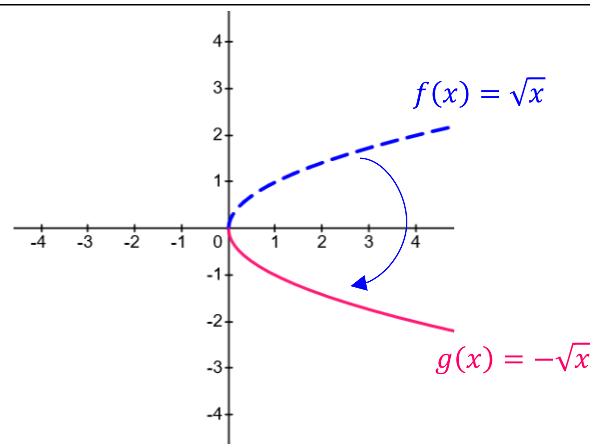


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**Reflect over the $y$-axis:**
(folding the graph at the $y$-axis)

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$g(x) = f(-x)$

$g(x) = \sqrt{-x}$

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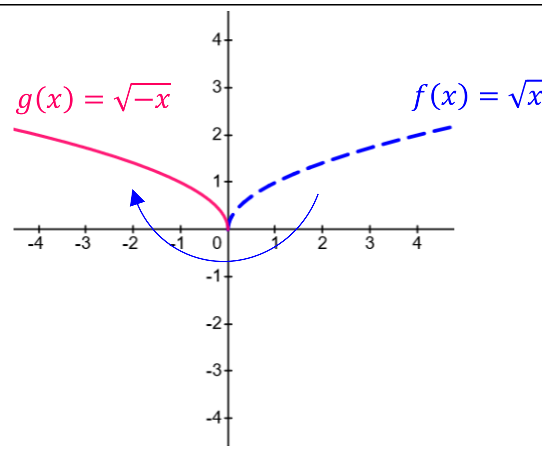


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**Stretched vertically by a factor of 2:**
(multiply each $y$-value by 2)

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$g(x) = 2f(x)$

$g(x) = 2\sqrt{x}$

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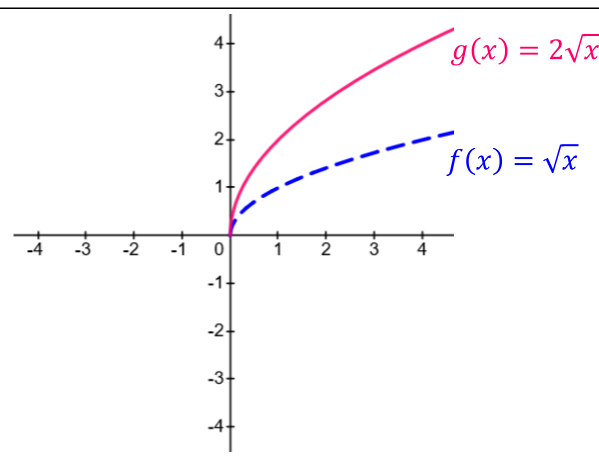


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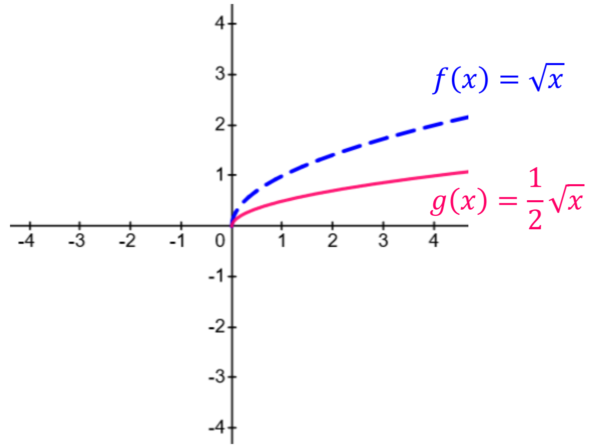
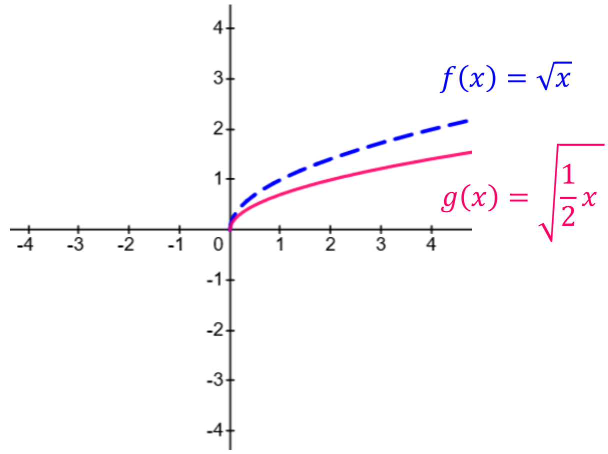
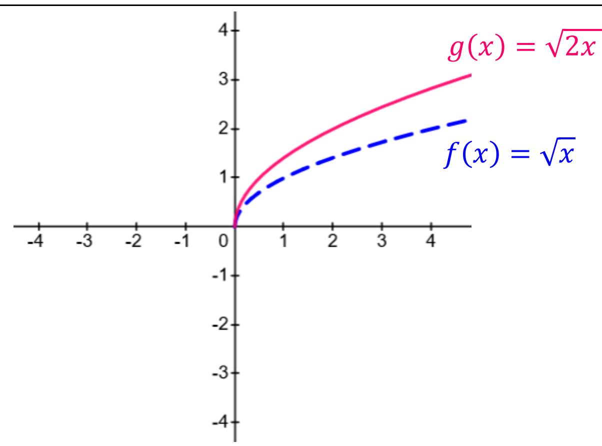
|---|---|---|
| **Compressed vertically by a factor of $\frac{1}{2}$:** (multiply each $y$-value by $\frac{1}{2}$) | $g(x) = \frac{1}{2}f(x)$ <br> $g(x) = \frac{1}{2}\sqrt{x}$ |   |
| **Stretch Horizontally by a factor of 2:** (multiply each $x$-value by 2) (same as dividing by $\frac{1}{2}$) | $g(x) = f\left(\frac{1}{2}x\right)$ <br> $g(x) = \sqrt{\frac{1}{2}x}$ |   |
| **Compress Horizontally by a factor of $\frac{1}{2}$:** (divide each $x$-value by 2) | $g(x) = f(2x)$ <br> $g(x) = \sqrt{2x}$ |   |

# Summary of Transformations

| **Transformation** | **Notation** | **Transformation of Point** |
|---|---|---|
| Vertical Translation:<br>Shift up $k$ units | $f(x) + k$ | $(x, y) \to (x, y + k)$ |
| Vertical Translation:<br>Shift down $k$ units | $f(x) - k$ | $(x, y) \to (x, y - k)$ |
| Horizontal Translation:<br>Shift left $h$ units | $f(x + h)$ | $(x, y) \to (x - h, y)$ |
| Horizontal Translation:<br>Shift right $h$ units | $f(x - h)$ | $(x, y) \to (x + h, y)$ |
| Reflection about the $x$-axis | $-f(x)$ | $(x, y) \to (x, -y)$ |
| Reflection about the $y$-axis | $f(-x)$ | $(x, y) \to (-x, y)$ |
| Vertical Stretch<br>by a factor of $a$ | $af(x)$<br>when $a > 1$ | $(x, y) \to (x, ay)$ |
| Vertical Compress<br>by a factor of $a$ | $af(x)$<br>when $0 < a < 1$ | $(x, y) \to (x, ay)$ |
| Horizontal Stretch<br>by a factor of $\frac{1}{b}$ | $f(bx)$<br>when $0 < b < 1$ | $(x, y) \to \left(\frac{x}{b}, y\right)$ |
| Horizontal Compress<br>by a factor of $\frac{1}{b}$ | $f(bx)$<br>when $b > 1$ | $(x, y) \to \left(\frac{x}{b}, y\right)$ |