

Transformations of Functions

Transformations of functions are functions made by the shifting, reflecting, or scaling of another function.

Example: We can use an example function of $f(x) = \sqrt{x}$ to illustrate all the types of transformations.

Transformation	Notation	Graph
Vertical shift 2 units up:	$g(x) = f(x) + 2$ $g(x) = \sqrt{x} + 2$	<p>A Cartesian coordinate system showing two curves. The x-axis ranges from -4 to 4, and the y-axis ranges from -4 to 4. A dashed blue curve represents the function $f(x) = \sqrt{x}$, starting at the origin (0,0) and increasing. A solid red curve represents the function $g(x) = \sqrt{x} + 2$, which is the dashed curve shifted 2 units upwards. A vertical blue arrow points upwards between the two curves at $x=2$.</p>
Vertical shift 2 units down:	$g(x) = f(x) - 2$ $g(x) = \sqrt{x} - 2$	<p>A Cartesian coordinate system showing two curves. The x-axis ranges from -4 to 4, and the y-axis ranges from -4 to 4. A dashed blue curve represents the function $f(x) = \sqrt{x}$, starting at the origin (0,0) and increasing. A solid red curve represents the function $g(x) = \sqrt{x} - 2$, which is the dashed curve shifted 2 units downwards. A vertical blue arrow points downwards between the two curves at $x=2$.</p>
Horizontal shift 2 units left:	$g(x) = f(x + 2)$ $g(x) = \sqrt{x + 2}$	<p>A Cartesian coordinate system showing two curves. The x-axis ranges from -4 to 4, and the y-axis ranges from -4 to 4. A dashed blue curve represents the function $f(x) = \sqrt{x}$, starting at the origin (0,0) and increasing. A solid red curve represents the function $g(x) = \sqrt{x + 2}$, which is the dashed curve shifted 2 units to the left. A horizontal blue arrow points to the left between the two curves at $y=1$.</p>

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<p>Horizontal shift 2 units right:</p> $g(x) = f(x - 2)$ $g(x) = \sqrt{x - 2}$		<p>The graph shows the function $f(x) = \sqrt{x}$ as a dashed blue curve starting at the origin (0,0) and increasing. A solid red curve, labeled $g(x) = \sqrt{x - 2}$, is shown shifted 2 units to the right. A blue arrow points from the point (0,0) on the dashed curve to the point (2,0) on the solid curve.</p>
<p>Reflection over the x-axis: (folding the graph at the x-axis)</p> $g(x) = -f(x)$ $g(x) = -\sqrt{x}$		<p>The graph shows the function $f(x) = \sqrt{x}$ as a dashed blue curve starting at the origin (0,0) and increasing. A solid red curve, labeled $g(x) = -\sqrt{x}$, is shown reflected across the x-axis. A blue arrow points from the point (0,0) on the dashed curve to the point (0,-1) on the solid curve.</p>
<p>Reflect over the y-axis: (folding the graph at the y-axis)</p> $g(x) = f(-x)$ $g(x) = \sqrt{-x}$		<p>The graph shows the function $f(x) = \sqrt{x}$ as a dashed blue curve starting at the origin (0,0) and increasing. A solid red curve, labeled $g(x) = \sqrt{-x}$, is shown reflected across the y-axis. A blue arrow points from the point (0,0) on the dashed curve to the point (-1,0) on the solid curve.</p>
<p>Stretched vertically by a factor of 2: (multiply each y-value by 2)</p> $g(x) = 2f(x)$ $g(x) = 2\sqrt{x}$		<p>The graph shows the function $f(x) = \sqrt{x}$ as a dashed blue curve starting at the origin (0,0) and increasing. A solid red curve, labeled $g(x) = 2\sqrt{x}$, is shown stretched vertically by a factor of 2. A blue arrow points from the point (0,0) on the dashed curve to the point (0,1) on the solid curve.</p>

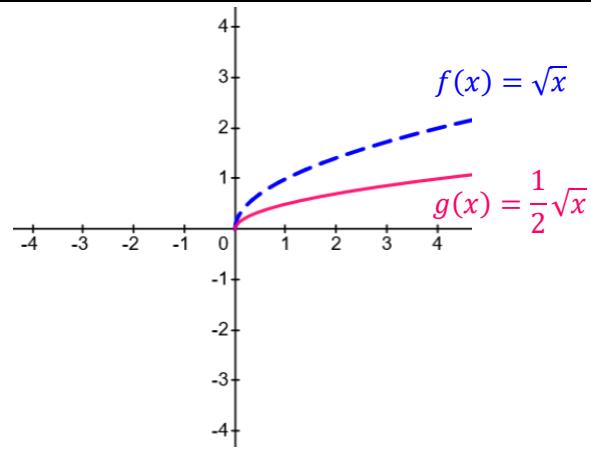
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**Compressed vertically
by a factor of $\frac{1}{2}$:**

(multiply each y -value by $\frac{1}{2}$)

$$g(x) = \frac{1}{2}f(x)$$

$$g(x) = \frac{1}{2}\sqrt{x}$$



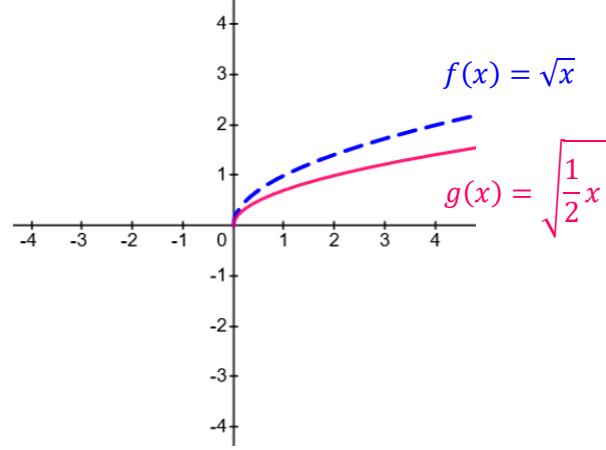
**Stretch Horizontally
by a factor of 2:**

(multiply each x -value by 2)

(same as dividing by $\frac{1}{2}$)

$$g(x) = f\left(\frac{1}{2}x\right)$$

$$g(x) = \sqrt{\frac{1}{2}x}$$

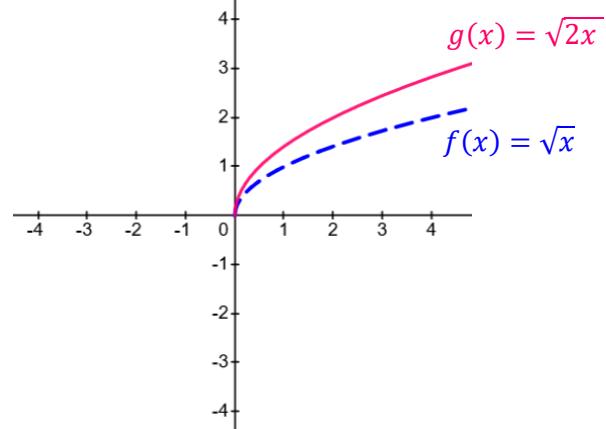


**Compress Horizontally
by a factor of $\frac{1}{2}$:**

(divide each x -value by 2)

$$g(x) = f(2x)$$

$$g(x) = \sqrt{2x}$$



Summary of Transformations

Transformation	Notation	Transformation of Point
Vertical Translation: Shift up k units	$f(x) + k$	$(x, y) \rightarrow (x, y + k)$
Vertical Translation: Shift down k units	$f(x) - k$	$(x, y) \rightarrow (x, y - k)$
Horizontal Translation: Shift left h units	$f(x + h)$	$(x, y) \rightarrow (x - h, y)$
Horizontal Translation: Shift right h units	$f(x - h)$	$(x, y) \rightarrow (x + h, y)$
Reflection about the x -axis	$-f(x)$	$(x, y) \rightarrow (x, -y)$
Reflection about the y -axis	$f(-x)$	$(x, y) \rightarrow (-x, y)$
Vertical Stretch by a factor of a	$af(x)$ when $a > 1$	$(x, y) \rightarrow (x, ay)$
Vertical Compress by a factor of a	$af(x)$ when $0 < a < 1$	$(x, y) \rightarrow (x, ay)$
Horizontal Stretch by a factor of $\frac{1}{b}$	$f(bx)$ when $0 < b < 1$	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$
Horizontal Compress by a factor of $\frac{1}{b}$	$f(bx)$ when $b > 1$	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$