Integration Properties

Property:	**Example:**
$\int kf(x)dx = k\int f(x)dx$	$\int 7x^3dx = 7\int x^3dx$
$\int (f(x)+g(x))dx = \int f(x)dx \;+\; \int g(x)dx$	$\int (2x^4+x^{-5})dx = \int 2x^4dx \;+\; \int x^{-5}dx$
$\int (f(x)-g(x))dx = \int f(x)dx \;-\; \int g(x)dx$	$\int (2x^4-x^{-5})dx = \int 2x^4dx \;-\; \int x^{-5}dx$

You will often use these fraction, root and exponent properties to simplify before integrating or finding antiderivative:

$\frac{a+c}{b} = \frac{a}{b}+\frac{c}{b}$ $\quad\sqrt{x} = x^{1/2}$ $\quad\sqrt[n]{x^m} = x^{m/n}$ $\quad x^{-1} = \frac{1}{x}$ $\quad\frac{1}{x^n} = x^{-n}$

Antiderivative/Integration Rules

Rule:	**Example:**
$\int kdx = kx + C$	$\int 8dx = 8x + C$
$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$	$\int x^6 dx = \frac{x^{6+1}}{6+1} + C = \frac{x^7}{7} + C \;= \frac{1}{7}x^7 + C$
$\int e^x dx = e^x + C$	
$\int e^{kx}dx = \frac{e^{kx}}{k} + C$	$\int e^{3x}dx = \frac{e^{3x}}{3} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int 5^x dx = \frac{5^x}{\ln 5} + C$
$\int \frac{1}{x}dx = \ln\lvert x\rvert + C$	

Example: Find the indefinite integral $\int \frac{2+x-x^3}{\sqrt{x}}dx$

Step 1: Simplify the integral

$$\int\left(\frac{2}{\sqrt{x}}+\frac{x}{\sqrt{x}}-\frac{x^3}{\sqrt{x}}\right)dx \;=\int\left(\frac{2}{x^{1/2}}+\frac{x}{x^{1/2}}-\frac{x^3}{x^{1/2}}\right)dx \;=\int\left(2x^{-1/2}+x^{1-1/2}-x^{3-1/2}\right)dx$$

$$=\int\left(2x^{-1/2}+x^{1/2}-x^{5/2}\right)dx$$

Step 2: Integrate

$$=\frac{2x^{-1/2+1}}{-\frac{1}{2}+1}+\frac{x^{1/2+1}}{\frac{1}{2}+1}-\frac{x^{5/2+1}}{\frac{5}{2}+1}+C \quad = \frac{2x^{1/2}}{\frac{1}{2}}+\frac{x^{3/2}}{\frac{3}{2}}-\frac{x^{7/2}}{\frac{7}{2}}+C$$

$$= 4x^{1/2}+\frac{2}{3}x^{3/2}-\frac{2}{7}x^{7/2}+C$$

Antiderivative/Integration Rules Involving Trigonometry

$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$
$\int \tan x\,dx = -\ln\lvert \cos x \rvert + C$ $= \ln\lvert \sec x\rvert + C$	$\int \cot x\,dx = \ln\lvert \sin x\rvert + C$
$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$	$\int \csc x\,dx = \ln\lvert \csc x - \cot x \rvert + C$
$\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$	$\int \frac{1}{\sqrt{a^2-u^2}}du = sin^{-1}\frac{u}{a} + C$

Example: Find the indefinite integral $\int \frac{\sin 2x}{\sin x}dx$

Step 1: Simplify the integral

Use trig identity $\sin 2x = 2\sin x\cos x$

$$\int \frac{2\sin x\cos x}{\sin x}dx = \int 2\cos x\,dx$$

Step 2: Integrate

$$\int 2\cos x\,dx = 2\sin x + C$$