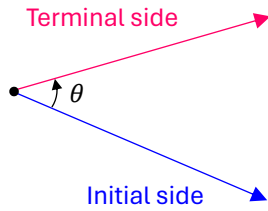


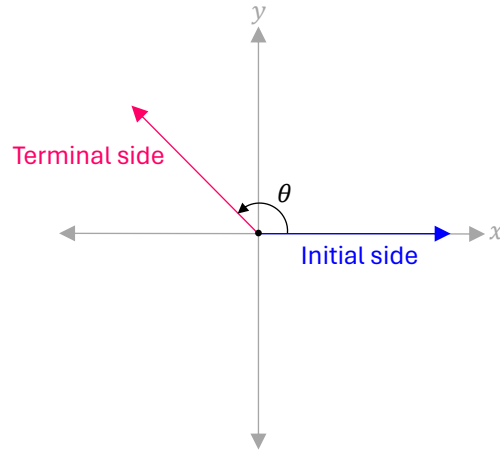
# Angles

**Initial side** – fixed side where angle starts.

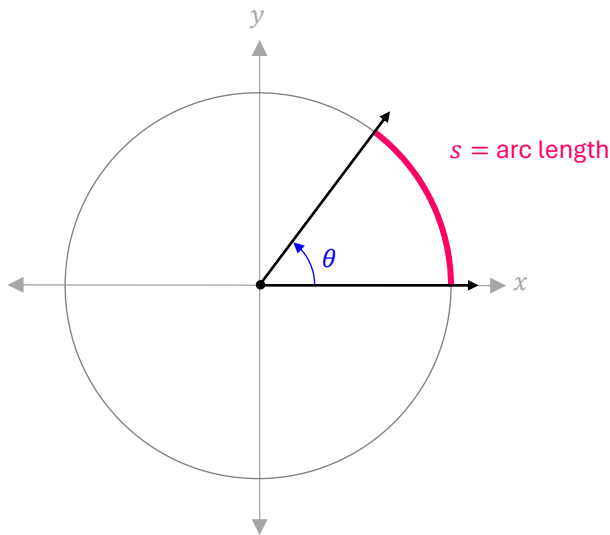
**Terminal side** – rotated side where angle ends.



**Standard Position** – initial side is on the positive  $x$ -axis.



**Central Angle** – angle whose vertex is at the center of a circle.



## Formulas:

where  $r$  = radius,  $\theta$  = angle in radians,  $t$  = time

<b>Arc length:</b>	$s = r\theta$
<b>Area of sector:</b>	$A = \frac{1}{2}r^2\theta$
<b>Linear speed:</b> (speed along straight path)	$v = \frac{s}{t}$
	$v = r\omega$
<b>Angular speed:</b> (speed along circular path)	$\omega = \frac{\theta}{t}$
	$\omega$ is in radians per unit time

## Measures of Angles

1 revolution around a circle =  $2\pi$  radians.

$2\pi$  radians =  $360^\circ$

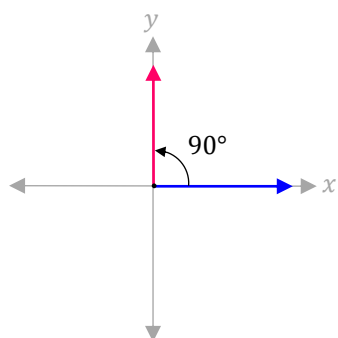
**Convert degrees to radian:** multiply by  $\frac{\pi}{180}$

**Example:**  $90^\circ = 90 \cdot \frac{\pi}{180} = \frac{\pi}{2}$  radians.

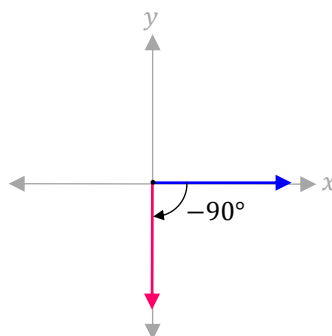
**Convert radian to degree:** multiply by  $\frac{180}{\pi}$

**Example:**  $\frac{\pi}{6}$  radians =  $\frac{\pi}{6} \cdot \frac{180}{\pi} = \frac{180}{6} = 30^\circ$

**Positive angles** move counterclockwise.

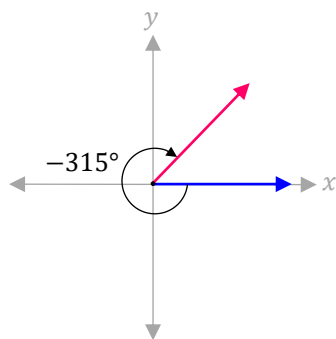
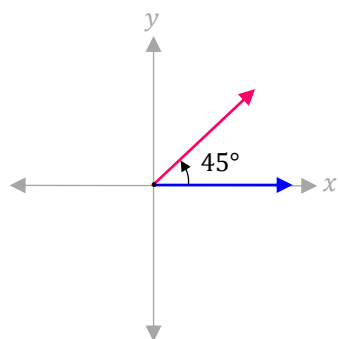


**Negative angles** move clockwise.



**Co-terminal angles** have the same initial and terminal side.

**Example:**  $45^\circ$  and  $-315^\circ$  are co-terminal angles.

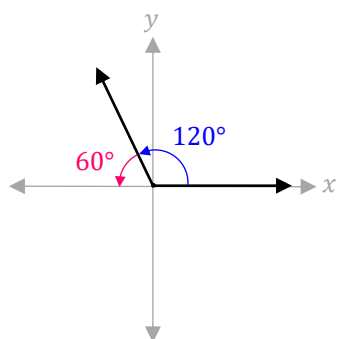


**Quadrantal angles:**

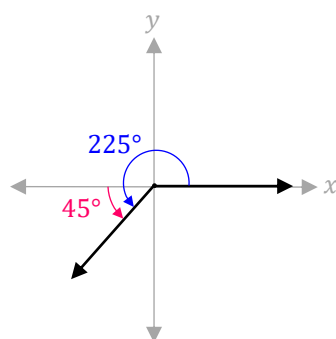
In degrees:	$90^\circ, 180^\circ, 270^\circ, 360^\circ \dots 90k$
In radian:	$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \dots \frac{\pi}{2}k$

For some angle  $\theta$  in standard position, its **reference angle** is the positive acute angle formed by the terminal side and the horizontal axis.

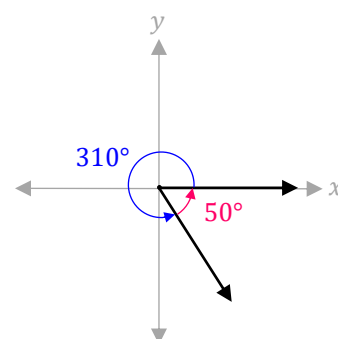
**Examples:**



$\theta = 120^\circ$   
Reference angle =  $180^\circ - 120^\circ = 60^\circ$



$\theta = 225^\circ$   
Reference angle =  $225^\circ - 180^\circ = 45^\circ$



$\theta = 310^\circ$   
Reference angle =  $360^\circ - 310^\circ = 50^\circ$