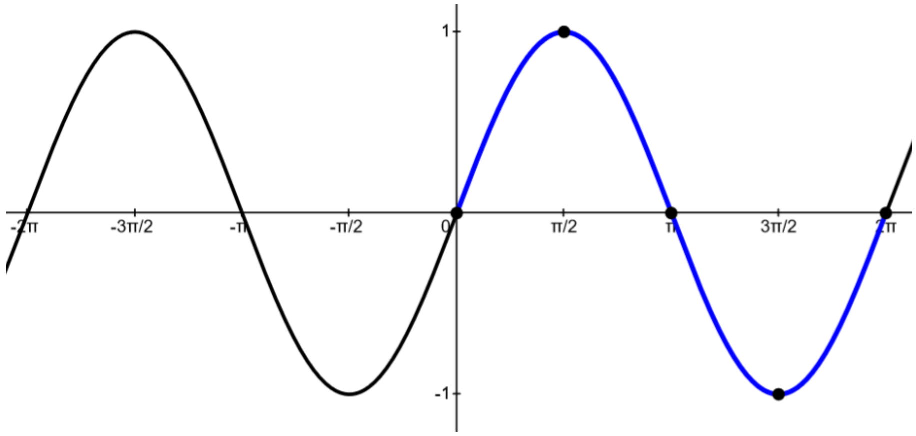
Graphing Sine and Cosine

You can plot the values of the quadrantal angles from the unit circle to graph 5 key points of one cycle of the function.

$f(\theta) = \sin(\theta)$

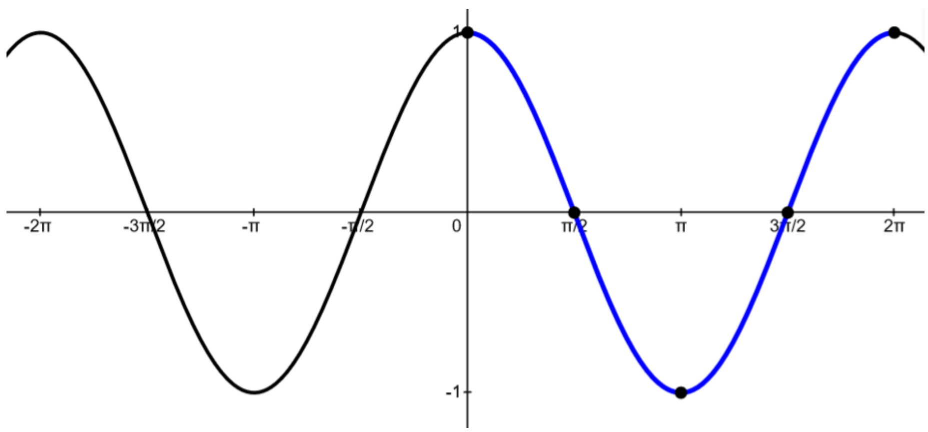
θ	$f(\theta)$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0



$\sin\theta = y$ Domain: $(-\infty, \infty)$ Range: $[-1, 1]$ Period: 2π

$f(\theta) = \cos(\theta)$

θ	$f(\theta)$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1



$\cos\theta = x$ Domain: $(-\infty, \infty)$ Range: $[-1, 1]$ Period: 2π

Transformations

$$y = A\sin(Bx - C) + D$$

- Amplitude (stretch/compression) $= |A|$ — $|A| = \text{amplitude} = \frac{\max - \min}{2} = \max - D = D - \min$
- Period $= \frac{2\pi}{B}$ $(B > 0)$ — Frequency (number of cycles per time): $F = \frac{1}{\text{period}}$
- Phase Shift (horizontal shift) $= \frac{C}{B}$ — Range $= -A + D$ and $A + D$
- Vertical shift (midline) $= D$ — $D = \text{midline} = \frac{\max + \min}{2}$

Example: Graph 1 period of the function $f(x) = 4\cos\left(2x + \frac{\pi}{3}\right) - 3$

Step 1: Identify the transformations and period.

$$y = A\cos(Bx - C) + D \quad \rightarrow \quad f(x) = 4\cos\left(2x - \left(-\frac{\pi}{3}\right)\right) - 3$$

$A = 4 \qquad B = 2 \qquad C = -\frac{\pi}{3} \qquad \frac{C}{B} = -\frac{\pi}{6} \qquad D = -3 \qquad$ Period $= \frac{2\pi}{2} = \pi$

Step 2: Start with the 5 key points for $\cos\theta$ for one period, then convert points.

$f(x) = \cos x$

x	y
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

New x-values:

Divide by B, then add $\frac{C}{B}$

$0/2 + \left(-\frac{\pi}{6}\right) = -\frac{\pi}{6}$

$\frac{\pi}{2}/2 + \left(-\frac{\pi}{6}\right) = \frac{\pi}{12}$

and so on.

Note also you can find
the change in x values by period$/4$
so the change in x is $\pi/4$

New y-values:
Multiply by A, then add D.

$1(4) - 3 = 1$

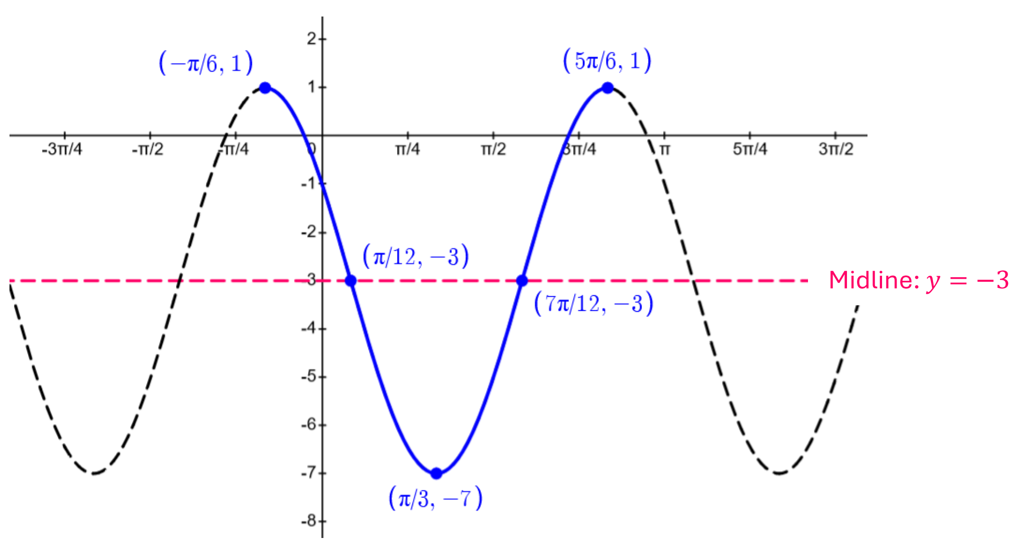
$0(4) - 3 = -3$

$-1(4) - 3 = -7$

$f(x) = 4\cos\left(2x + \frac{\pi}{3}\right) - 3$

x	y
$-\frac{\pi}{6}$	1
$\frac{\pi}{12}$	-3
$\frac{\pi}{3}$	-7
$\frac{7\pi}{12}$	-3
$\frac{5\pi}{6}$	1

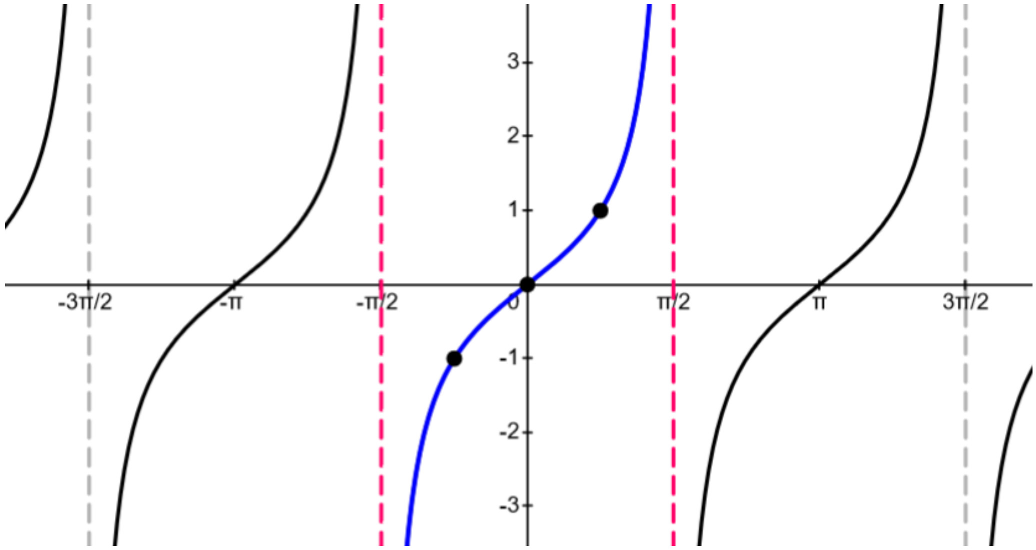
Step 3: Plot points and graph:



Graphs of the Other Trigonometric Functions

$f(\theta) = \tan(\theta)$

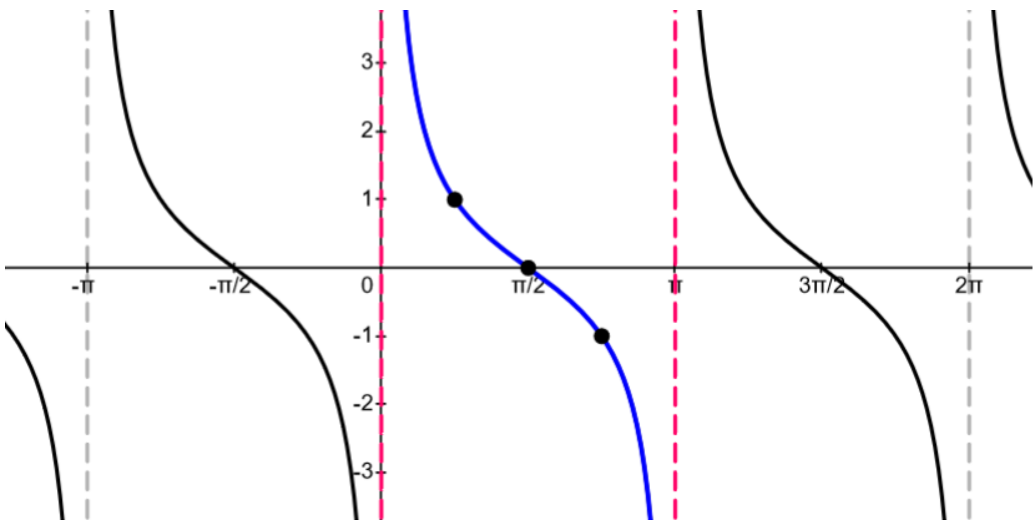
θ	$f(\theta)$
$-\frac{\pi}{2}$	undefined
$-\frac{\pi}{4}$	-1
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	undefined



$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$ Asymptotes: $x = \frac{\pi}{2} + \pi k$ Period: π

$f(\theta) = \cot(\theta)$

θ	$f(\theta)$
0	undefined
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	undefined

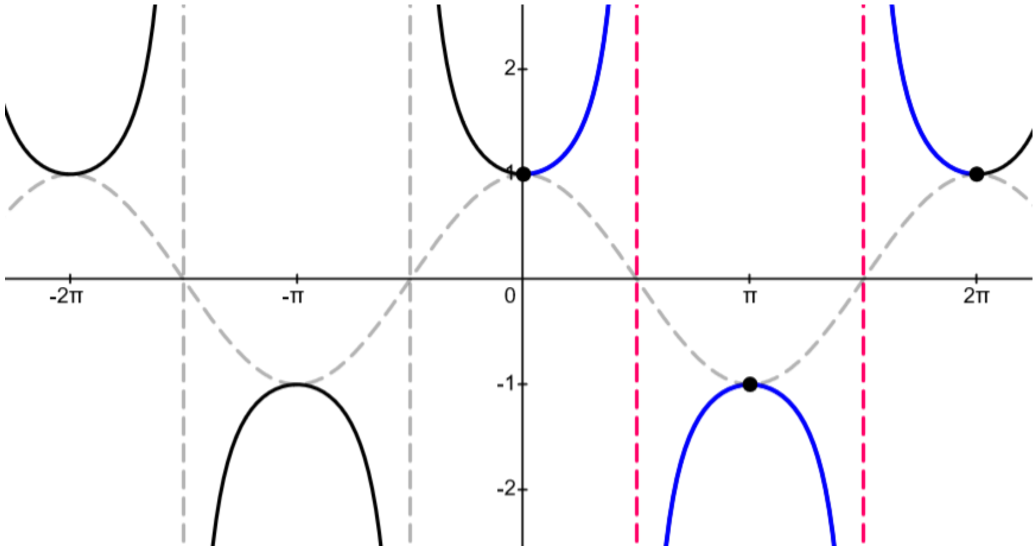


$\cot\theta = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$ Asymptotes: $x = \pi k$ Period: π

Graphs of the Other Trigonometric Functions

$f(\theta) = \sec(\theta)$

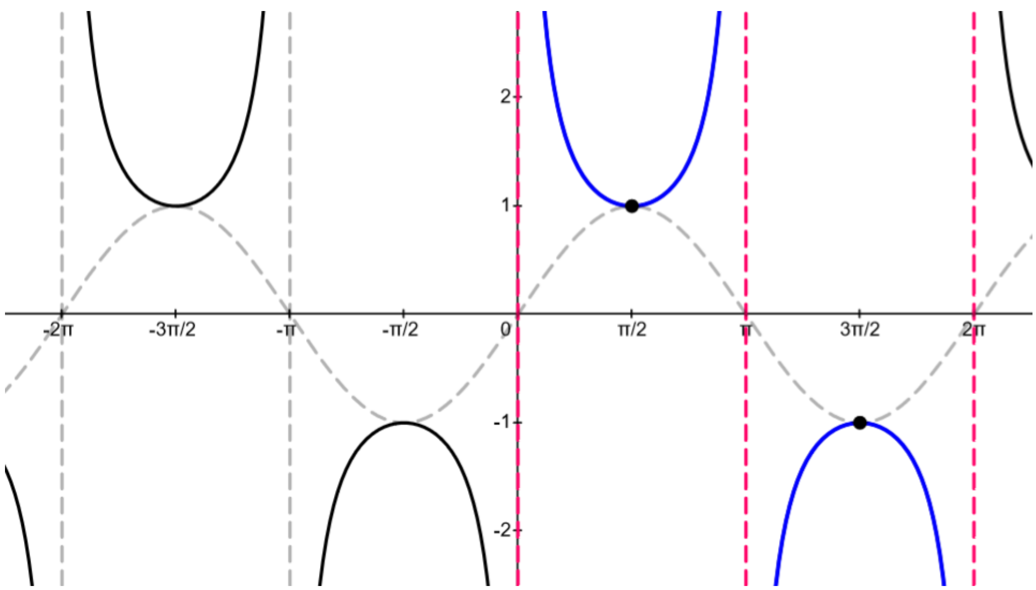
θ	$f(\theta)$
0	1
$\frac{\pi}{2}$	undefined
π	-1
$\frac{3\pi}{2}$	undefined
2π	1



$\sec\theta = \frac{1}{x} = \frac{1}{\cos\theta}$ Asymptotes: $x = \frac{\pi}{2} + \pi k$ Period: 2π

$f(\theta) = \csc(\theta)$

θ	$f(\theta)$
0	undefined
$\frac{\pi}{2}$	1
π	undefined
$\frac{3\pi}{2}$	-1
2π	undefined



$\csc\theta = \frac{1}{y} = \frac{1}{\sin\theta}$ Asymptotes: $x = \pi k$ Period: 2π
