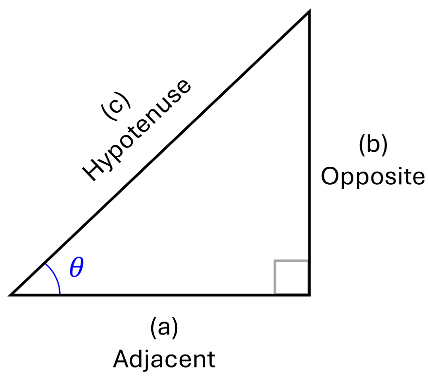
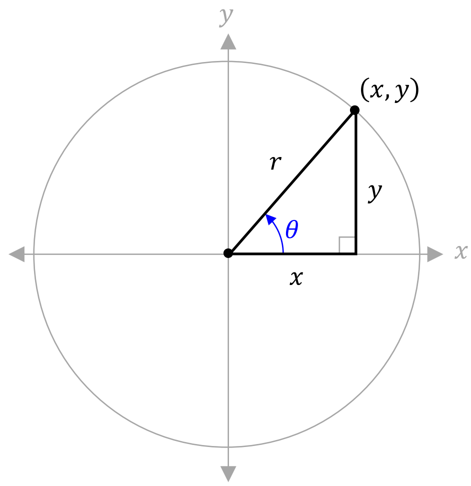
Right Triangle Trigonometry





Pythagorean Theorem:	this can also be shown as:
$a^2 + b^2 = c^2$ solved for c: $c = \sqrt{a^2 + b^2}$	$x^2 + y^2 = r^2$ solved for r: $r = \sqrt{x^2 + y^2}$

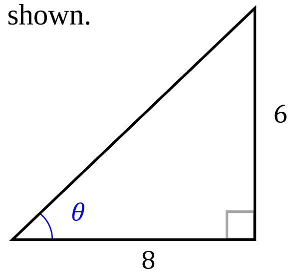
Trigonometric Functions

– functions that have the input as an angle of a right triangle, and output as a ratio of two sides of a right triangle.

Soh	**Cah**	**Toa**
$\text{sine} = \dfrac{\text{opposite}}{\text{hypotenuse}}$	$\text{cosine} = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	$\text{tangent} = \dfrac{\text{opposite}}{\text{adjacent}}$

Trig functions:	**side ratios**	**in terms of x, y and r**	**reciprocal identities**	**other identities**
Sine	$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$	$\sin\theta = \dfrac{y}{r}$	$\sin\theta = \dfrac{1}{\csc\theta}$	
Cosine	$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$	$\cos\theta = \dfrac{x}{r}$	$\cos\theta = \dfrac{1}{\sec\theta}$	
Tangent	$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$	$\tan\theta = \dfrac{y}{x}$	$\tan\theta = \dfrac{1}{\cot\theta}$	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
Cosecant	$\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$	$\csc\theta = \dfrac{r}{y}$	$\csc\theta = \dfrac{1}{\sin\theta}$	
Secant	$\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$	$\sec\theta = \dfrac{r}{x}$	$\sec\theta = \dfrac{1}{\cos\theta}$	
Cotangent	$\cot\theta = \dfrac{\text{adj}}{\text{opp}}$	$\cot\theta = \dfrac{x}{y}$	$\cot\theta = \dfrac{1}{\tan\theta}$	$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Example 1: Find the exact values of the six trig functions of the angle θ shown.



Step 1: Solve for the hypotenuse side using the Pythagorean theorem.

$$\text{adjacent} = 8, \ \text{opposite} = 6$$

$$a^2 + b^2 = c^2 \quad \Rightarrow \quad 8^2 + 6^2 = c^2 \quad \Rightarrow \quad 64 + 36 = c^2 \quad \Rightarrow \quad 100 = c^2 \quad \Rightarrow \quad \sqrt{100} = \sqrt{c^2}$$

$$\text{hypotenuse} = c = 10$$

Step 2: Use “Soh Cah Toa” and/or identities to solve for the trig functions.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} = \frac{4}{5} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} = \frac{3}{4}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{10}{6} = \frac{5}{3} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{10}{8} = \frac{5}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{8}{6} = \frac{4}{3}$$

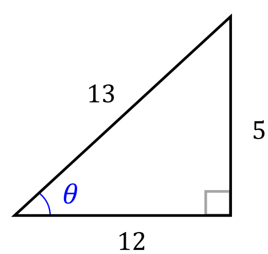
Example 2: Find the other five trig functions of the angle θ, given $\sec\theta = \frac{13}{12}$

Step 1: Use the reciprocal identity $\cos\theta = \frac{1}{\sec\theta}$ or $\sec\theta = \frac{\text{hyp}}{\text{adj}}$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13} = \frac{\text{adj}}{\text{hyp}}$$

Step 2: Solve for the opposite side using the Pythagorean theorem.

$$a^2 + b^2 = c^2 \quad \Rightarrow \quad 12^2 + b^2 = 13^2 \quad \Rightarrow \quad b^2 = 169 - 144 \quad \Rightarrow \quad b^2 = 25 \quad \Rightarrow \quad b = 5$$



Step 3: Use “Soh Cah Toa” and/or identities to solve for the trig functions.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{5}{12}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{13}{5} \qquad \sec\theta = \frac{13}{12} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$$